APPLICABILITY OF DETERMINISTIC INVENTORY MODELS TO SOLVE STOCHASTIC INVENTORY SYSTEMS

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APPLICABILITY OF DETERMINISTIC INVENTORY MODELS

TO SOLVE STOCHASTIC INVENTORY SYSTEMS

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TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The objective of this work is to determine the effectiveness of a deterministic inventory model in representing and solving an inventory system with stochastic elements. A deterministic model with infinite resupply rate in which backorders are allowed was compared with two stochastic models: the stochastic lot size reorder point model in which a transaction reporting system must be used and the stochastic fixed cycle model in which periodic review may be used.

To measure the effectiveness of the deterministic model a variable called Percent Deviation was defined. This variable determines the percent cost increase incurred when the deterministic policy is used to operate the stochastic system instead of the optimal stochastic policy.

Two computer programs for the UNIVAC 1108 written in Fortran V were prepared to perform the necessary calculations. Also a discussion of the effects of the cost of operating the policies is presented.

From the results of the analysis it can be concluded that the deterministic policy will, in many cases, be a very good approximation to use to operate a given stochastic inventory system even though the assumptions on which it was developed are, in part, violated.

CHAPTER I

INTRODUCTION

Regardless of how small an enterprise is, it will always have to maintain some type of items in inventory such as finished goods, work in process, raw materials, etc. This might be due to any of the following: variation of demand rate during the year, the necessity for buffer stock to absorb variations of production rate, changes in prices of raw material at different periods of time, or other factors. Thus, every manager in one way or another will be involved with inventory problems. Because of this, in recent years inventory theory has been extensively studied. Also, as Hadley and Whitin (12) point out, inventory models have been studied by an increasing number of investigators just because they present interesting mathematical problems.

When establishing an inventory policy two questions must be answered: when to order and how much to order. Once the policy has been established there remains the task of controlling the inventory system. Since the initial developments of inventory theory in 1915 when the Economical Order Quantity (EOQ) formula was developed by F. W. Harris to our days, much has been done in developing it. Today, inventory theory provides inventory managers with a great number of models for solving inventory systems, each of which is directed toward a specific situation. As the models represent more accurately real life situations, they become more difficult to solve, implement, and operate. Some models require the use of high speed computers and highly trained personnel with mathematical backgrounds to be solved. Even the task of selecting the model that would give the most economical policy to solve the inventory system is not an easy one.

For these reasons many small companies do not use any of the available inventory models to solve their inventory problems. As stated by Whitin (35) a great percentage of the largest companies in the United States do not use any form of operation analysis in controlling their inventories. Instead, intuition, based on experience, is used to select their inventory policies.

No matter what method a manager uses to control his inventories, he must previously determine some characteristics of the system such as demand pattern, cost parameters, lead times, etc. Provided that these characteristics are determined, he must then select the model that will give him the most economical policy to solve his inventory system. This work is intended to give some aid in this selection process.

Purpose of the Research

The objective of this work is to determine the conditions under which a deterministic inventory policy would be a good approximation to use for solving an inventory system with stochastic parameters.

In most real world problems the inventory manager deals with systems which involve stochastic demands and/or stochastic procurement lead times. Even though the deterministic models were developed for a situation in which these parameters were deterministic, they still may be applicable in situations which differ from the ideal one.

It will be shown that even if complex stochastic models represent the system more accurately, the cost reduction gained by their use may not be enough to make them the most economical ones. This is because an additional cost for operating the system is incurred when a stochastic model is used, which is not incurred when a deterministic model is used. In general this cost is not included in determining the optimal stochastic policy, but must be considered whenever a comparison between policies is made.

Literature Review

Even though the literature on inventory theory is vast, not many comparisons between models were found. In general the comparisons found were between deterministic models in deterministic situations or stochastic models in stochastic situations. Several works that deal with deterministic situations will be presented first.

A comparison that has been studied by several authors is the one between the EOQ approach versus the Wagner-Whitin (w-w) dynamic formulations in the case of deterministic situations. Kaiman (18,19,20) attempts to demonstrate that the w-w formulation will give a more economical operation of inventory systems but finally concludes that each formulation is suited for a particular demand pattern. He also gives criteria for when to switch from one formulation to the other.

On the same subject Tuite and Anderson (33) claimed that the comparison done by Kaiman had errors in it. They argue that in the w-w formulation Kaiman did not consider the holding cost for the "period of use." They conclude as Kaiman that both methods are suited for extreme cases and also that the w-w formulation effectiveness depends on the length of the time period chosen.

Silver and Meal (29) developed a modification of the simple Wilson formula so that it could take into account variations of the demand rate. They compared their formulation with the EOQ and w-w methods and conclude that their formulation will give significant cost reduction as compared to the two other formulations.

Gorestein (11) shows that the EOQ and w-w are not comparable. He studies and compares the papers of Kaiman, Tuite and Anderson, and Silver and Meal. He concludes that both formulations, in a sense, are not comparable. Each is valid if the assumptions on which they were developed hold.

Eilon and Elmaleh (8) compared the performance of five inventory control models under stochastic demand patterns. The five models compared were: i) the (R,r) model or two bin policy; ii) the (R,T) model or fixed cycle periodic review model; iii) the (Q,T) model which is exactly as the (R,T) model but instead of ordering up to a level R, a fixed quantity Q is ordered every T units of time; iv) the (r,R,T) model where a review takes place every T units of time and an order is placed to bring the inventory position up to R. However, if the inventory position was previously less than the reorder point, an order is placed to bring it up to R.; v) the (r,Q,T) is the same as the previous (iv) but here a fixed quantity Q is ordered every time a replenishment is made. Eilon and Elmaleh compared the five models under different demand distributions and with normally distributed lead times where backorders were allowed. They conclude that the best policy was the (r,R,T) with respect to average stock level, total cost incurred, and other factors which were compared.

Hadley and Whitin (12) present and discuss several relevant characteristics of the models being studied in their book. They also compare periodic review and transaction reporting systems. They conclude that no general statement can be made as to whether one of the two pure inventory control systems is preferred.

Gallegher (10) made a comparison of two periodic review inventory models, the (R,T) and the (nQ,r,T). He assumes that demands are generated by a Poisson or Stuttering Poisson Process, backorders are allowed, and lead times are constant. He concludes that the (R,T) model has a greater applicability than seems possible at first glance. Also he concludes that, if the probability of placing an order at every review is high and the ordering cost is lower than the review cost, the (R,T) model should be preferred over the (nQ,r,T) model.

Newberry (26) compares four stochastic inventory models. He considers demand and lead time as random variables and the process generating them does not change over time. The models compared were the (Q,r), (R,T), (R,r), and (r,R,T). He uses as measures of effectiveness the probability of or more shortages, the expected number of shortages, and several others. He presents a procedure to compare these models for any particular situation and recommends possible new studies.

Zimmermann (37) compares the two main inventory control systems; that is, transaction reporting and periodic review. He concludes after an analytic comparison that in practice no pure system can be considered optimal. Transaction reporting will be more appropriate when: i) the number of transactions is low compared to annual usage, ii) the cost of processing a transaction is very low compared to the ordering cost, iii) high price per item, iv) high carrying charge, v) highly variable demand pattern, and vi) high protection against stockout is needed.

Besides these types of comparisons, a great number of articles have been written on the effects that cost structure, different demand distributions, and other factors have on the models.

Foster, Rosenhead, and Siskind (9) studied the effect that different types of demand distributions had on the (R,T) model. They considered the case in which all unsatisfied demand was lost. In their comparison they used the Poisson, Stuttering Poisson, and Gamma distributions. They concluded that the demand type had a sizeable effect on the optimal policy obtained from solving the model.

Sinha and Gupta (31) studied the sensitivity of the inventory models to the form of the cost function for surplus and shortages and also to different types of demand distributions. They conclude that the models are sensitive to the dimensions and form of the cost functions and parameters and also to the demand distribution being used.

In Chapter II of this work a description of the models used in the study is presented. Chapter III deals with the solution procedure and Chapters IV and V present the results and conclusions of the study.

CHAPTER II

MODELS USED IN STUDY

As stated previously, the purpose of this work is to compare the performance of stochastic and deterministic inventory models in approximating stochastic inventory systems. The comparison will be based on the economic tradeoffs that are involved when either one of the two types of models is applied to solve a stochastic situation. The characteristics of the inventory system assumed for the study are:

1. The number of demands for a given period is normally distributed.
2. The process generating demands does not change over time.
3. The procurement lead time is constant.
4. All variables can be treated as continuous.
5. The system stocks a single item in a single location.
6. The item never becomes obsolete.
7. All unsatisfied demands will be backordered.
8. The system has infinite resupply rate.

A deterministic inventory model with infinite rate of resupply which allows backorders was compared with two stochastic models. The two models are the stochastic lot size reorder point model that will be called the (Q,r) model and the stochastic fixed cycle model that will be called the (R,T) model. The (Q,r) model was chosen because it requires the use of transaction reporting and the (R,T) model because it is the most widely used operating doctrine for periodic review systems.

The models and solution procedures were selected from the ones presented by Hadley and Whitin (12).

Deterministic Model

For this study the deterministic model with infinite rate of resupply and where backorders are allowed was used. This model is commonly known and can be found in most books that deal with inventory theory. A detailed presentation of the development of the model will not be given here. For a detailed development of the model the reader may refer to Hadley and Whitin (12).

Figure 1 shows a graphical illustration of the system where Q is the fixed reorder quantity, S is the allowed number of backorders, T is the cycle time, T_1 is the time the system has items in stock, T_2 is the time the system is out of stock, τ is the procurement lead time, and r is the reorder point based on the inventory position.

The assumptions on which the model was developed are:

1. Demand is deterministic, constant, and equal to λ units per year.

2. The procurement lead time is constant and equal to τ years.

3. In determining the optimal deterministic policy the cost of operating the inventory system will not be considered because the optimal policy is independent of this cost.

4. The cost of each item is constant, equal to C dollars per unit, and independent of the quantity ordered and the reorder rule.

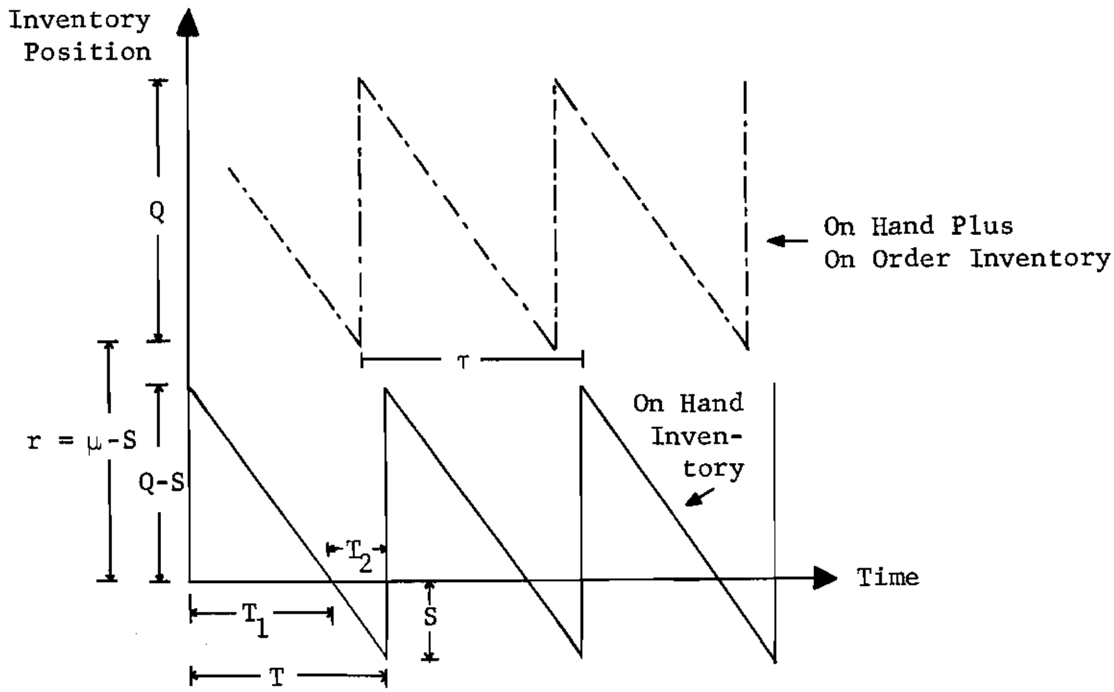


Figure 1. Graph of Deterministic Policy

5. The inventory carrying charge is constant and equal to I dollars per dollar-year invested in inventory.

6. The cost of a backorder has the form $\pi + \hat{\pi} t$ where π is the cost in dollars per unit backordered and $\hat{\pi}$ is the cost in dollars per unit year of backorder.

7. The cost of placing an order is constant, independent of the quantity ordered, and equal to A dollars per order.

8. The reorder quantity will be always Q units.

9. The number of backorders on hand when the procurement arrives will be S units. S will always be less than Q.

10. The reorder point r will be given in terms of the on hand plus on order quantity, i.e., the inventory position.

Based on these assumptions and on the characteristics of the inventory system, the model is used to find an optimal deterministic policy. To determine this policy the partial derivatives with respect to Q and S of the annual total cost equation are found and set equal to zero. Then both equations are solved in terms of S and the following equation is developed to determine the optimal policy.

$$S^2(\hat{\pi} IC - \hat{\pi}^2) + 2\lambda\pi\hat{\pi}S - (\pi\lambda)^2 - 2\lambda AIC = 0 \qquad (1)$$

where all the terms are as previously defined. Special cases may arise depending on the structure of the backorder cost.

<u>Case I</u>

If $\hat{\pi} = 0$ and if $\pi \neq 0$ then equation (1) reduces to

$$\pi\lambda = \sqrt{2\lambda AIC} \tag{2}$$

The right side of equation (2) is the total annual cost for the deterministic case where backorders are not allowed. This cost will be called KW. In general, equation (2) will not hold and so no value of S in the interval $0 < S < \infty$ will solve equation (1). That is, the optimal number of backorders for the inventory system will be on the boundaries of the interval. To find which is the optimal value of S we compare $\pi\lambda$ with KW. If $\pi\lambda$ is greater than KW, then the optimal value of S is $S^* = 0$, and the optimal Q will be given by

$$Q^* = \sqrt{\frac{2\lambda A}{IC}} \tag{3}$$

This value of Q is the optimal reorder quantity for the deterministic case where backorders are not allowed. It is known in literature as Wilson lot size, Q.W. For this case the optimal total annual cost will be KW. In the case that $\pi\lambda$ is less than KW, the optimal value of S is $S^* = \infty$ and then no inventory system exists because it will be cheaper to have all demands backordered than to operate the inventory system.

When equation (2) holds any value of S in the interval $0 < S < \infty$ will solve the system. The proof of this property is shown in Appendix I. The value of the optimal Q will depend on the value of S chosen and will be given by

$$Q^* = \frac{\pi\lambda}{IC} + S^* \tag{4}$$

and the total annual cost will be again KW.

Case II

If $\hat{\pi} \neq 0$ and $\pi = 0$, the model gives as the optimal values of S and Q

$$S^* = \sqrt{\frac{2\lambda AIC}{\pi(\hat{\pi} + IC)}} = \frac{KW}{\sqrt{\hat{\pi}(\hat{\pi} + IC)}} \tag{5}$$

$$Q^* = \sqrt{\frac{\hat{\pi} + IC}{\hat{\pi}}} \cdot \sqrt{\frac{2\lambda A}{IC}} = \sqrt{\frac{\hat{\pi} + IC}{\hat{\pi}}} \cdot QW \tag{6}$$

In this case unless $\hat{\pi} = \infty$, S will always be greater than zero. Thus for normal operating conditions it will always be optimal to incur some backorders. The annual total cost is given by

$$K = \frac{A\lambda}{Q^*} + \frac{IC}{2Q^*}(Q^* - S^*)^2 + \left(\frac{\pi S^{*2}}{2}\right) \times \frac{1}{Q^*} \tag{7}$$

Case III

If π and $\hat{\pi}$ are different than zero, the solution to equation (1) will give as the optimum number of backorders

$$S^* = \frac{-\pi\lambda + \sqrt{(2\lambda AIC)\left(1 + \frac{IC}{\hat{\pi}}\right) - \frac{IC}{\hat{\pi}}(\lambda\pi)^2}}{(\hat{\pi} + IC)} \tag{8}$$

and for the optimum reorder quantity

$$Q^* = \sqrt{\frac{\hat{\pi} + IC}{\hat{\pi}}} \sqrt{\frac{2\lambda A}{IC} - \frac{(\pi\lambda)^2}{IC(IC + \hat{\pi})}} \tag{9}$$

In the event that the value of S obtained from (8) is negative, then S^* is made equal to zero. Equation (9) holds only if S^* is greater than zero; otherwise Q^* will be QW and the total annual cost will be KW. If the value of S^* was greater than zero, then the total annual cost will be given by equation (7).

For all cases the reorder point r given in terms of the inventory position, is given by

$$r = \mu - S \tag{10}$$

where μ is the deterministic lead time demand calculated by

$$\mu = \tau \times \lambda \tag{11}$$

where τ is the procurement lead time. The value of the reorder point may be negative.

Stochastic Lot Size Reorder Point Model

The development of the model and solution procedure is due to Hadley and Whitin (12). Figure 2 gives a graphical representation of the model where Q is the fixed reorder quantity, r is the optimal reorder point based on the inventory position, and τ is the constant procurement lead time.

The assumptions of the model are:

1. Demands are normally distributed random variable with a mean of λ units per year. The process generating demands does not change with time.

2. Transaction reporting will be used to control the system.

3. Demands at different periods of time are independent random

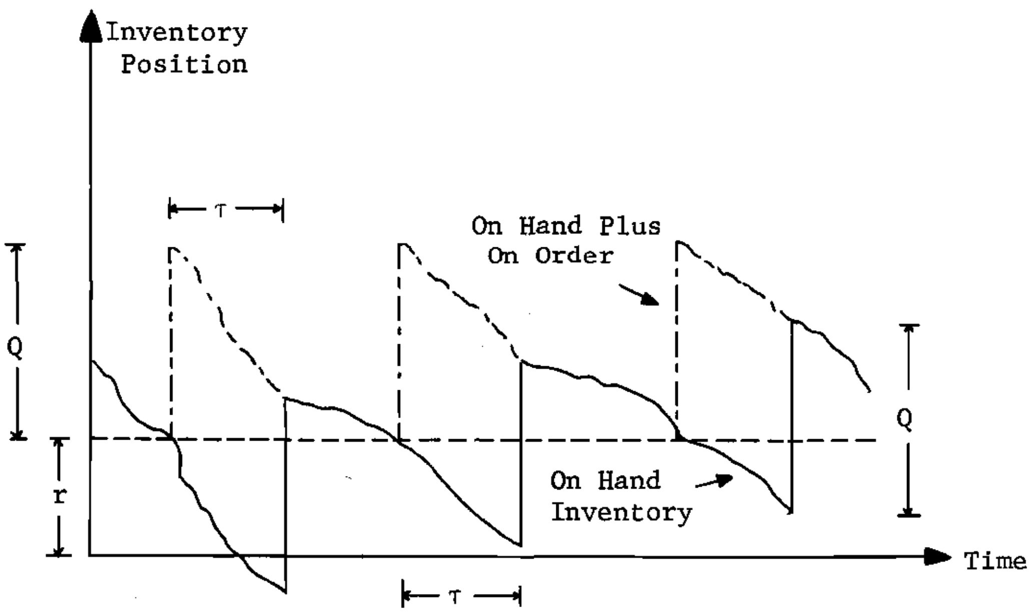


Figure 2. Graph of Stochastic (Q,r) Policy

variables.

4. The variance of the demand distribution will be considered to be independent of the rest of the parameters of the system.

5. One unit will be demanded every time a demand occurs.

The rest of the assumptions of the deterministic model regarding the parameters and cost structure will hold for this case. Based on these assumptions and the characteristics of the situation being modeled, the exact formulation model determines the following formulas to calculate the optimal policy.

The expected number of backorders incurred per year is

$$E(Q,r) = \frac{\lambda}{Q}\,[\alpha(r) + \alpha(r+Q)] \tag{12}$$

where

$$\alpha(v) = \sigma\phi\left(\frac{v-\mu}{\sigma}\right) - (v-\mu)\Phi\left(\frac{v-\mu}{\sigma}\right) \tag{13}$$

where $\phi(X)$ is the probability density function of a standard normal random variable X and $\Phi(X)$ is the complementary cumulative distribution function of the same random variable. Also, μ will be the expected lead time demand and σ will be the standard deviation of the demand during lead time.

The expected unit-years of backorder will be given by

$$B(Q,r) = \frac{1}{Q}\,[\beta(r) - \beta(r+Q)] \tag{14}$$

where

$$\beta(v) = \frac{1}{2}\left[\sigma^2 + (v-\mu)^2\right] \cdot \Phi\left(\frac{v-\mu}{\sigma}\right) - \frac{\sigma}{2}\,(v-\mu)\;\phi\left(\frac{v-\mu}{\sigma}\right) \tag{15}$$

Finally, the expected on-hand inventory will be given by

$$D(Q,r) = \frac{Q}{2} - r - \mu + B(Q,r) \tag{16}$$

where all the terms are as defined previously.

Because an order is placed after every Q demands have occurred and λ is the mean demand rate per year, the number of orders placed by year will be λ/Q. Combining all terms of the expected total cost annual cost gives

$$K \doteq \frac{\lambda}{Q} A + IC\ D(Q,r) + \pi E(Q,r) + \hat{\pi} B(Q,r) \tag{17}$$

As pointed out by Hadley and Whitin (12), this cost expression is not convex due to the terms $\alpha(r+Q)$ and $\beta(r+Q)$ of equations (12) and (14). These terms will be significant only if there exists a positive probability that lead time demand will be bigger than Q+r. In this case the reorder quantity would not be sufficient to remove all the backorders. In practice, this would only be optimal if the backorder costs were negligible, but this in general is not true. So, in practice these terms can be deleted and then the expected total annual cost is convex. Under this assumption, the following expressions can be solved simultaneously to find the optimal policy

$$Q = \sqrt{\frac{2\lambda\ [A+\pi\alpha(r)] + 2(\hat{\pi}+IC)\ \beta(r)}{IC}} \tag{18}$$

$$QIC = [\pi\lambda - (\hat{\pi}+IC)(r-\mu)] \cdot \Phi\left(\frac{r-\mu}{\sigma}\right) + \sigma(\hat{\pi}+IC)\ \phi\left(\frac{r-\mu}{\sigma}\right) \tag{19}$$

This will be the case used for this study. To find the optimal policy an iterative procedure will be used. First Q is set equal to the value of QW found from the deterministic part and substituted into equation (19). The value of r that satisfies this equation is determined; call it r_1. Substitution of r_1 into equation (18) gives a new value of Q; call it Q_1. With this new value and equation (19) a new value of r is found. This procedure is continued until no change in Q or r is obtained. This method has been proven to converge very rapidly as stated by Hadley and Whitin (12) and is suitable for programming on a digital computer.

Stochastic Fixed Cycle, Periodic Review Model

Again only a discussion of the model assumptions and solution procedures will be presented. The detailed development of the model is presented by Hadley and Whitin (12). Figure 3 gives a graphical representation of the system where T is the fixed interview time, τ is the fixed procurement lead time, and R is the "order up to" level given in terms of the inventory position.

The assumptions of the model are:

1. The number of demands is a normally distributed random variable with a mean of λ units per year. The process generating demands does not change with time.

2. The cost of making a review is J dollars per review and it is independent of the optimal policy.

3. An order is placed at every review and enough quantity is ordered to bring the inventory position up to a level R.

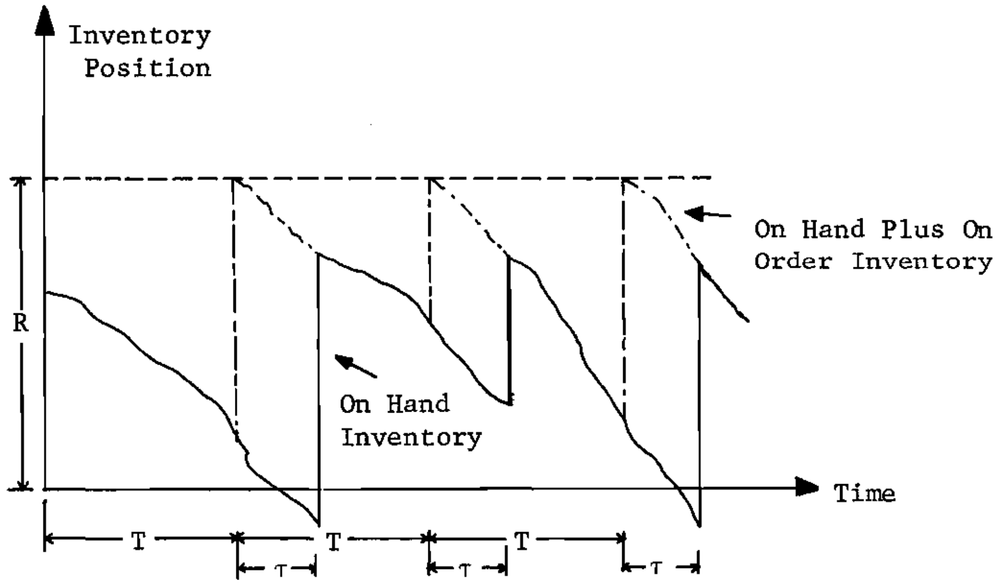


Figure 3. Graph of Stochastic (R,T) Policy

4. A review will be made every T years. This value is called interview time.

5. The "order up to" level, R, is given in terms of the inventory position.

6. For an interval of time, t, the mean of the normal distribution will be λt and the standard deviation will be $\sqrt{Dt}$.

7. Demands in different periods of time are independent random variables.

The assumptions on the cost structure and lead time remain the same as in the previous two models. With these model assumptions and the characteristics of the inventory system, the model uses the following formulas for each of the relevant terms of the policy.

The expected number of backorders per year is given by

$$E(R,T) = \frac{1}{T}\left\{\sqrt{D(\tau+T)}\cdot \phi\left[\frac{R-\lambda(\tau-T)}{\sqrt{D(\tau+T)}}\right] - [R-\lambda(\tau+T)]\,\Phi\left[\frac{R-\lambda(\tau+T)}{\sqrt{D(\tau+T)}}\right]\right. \tag{20}$$

$$\left. - \sqrt{D\tau}\;\phi\left(\frac{R-\lambda\tau}{\sqrt{D\tau}}\right) + (R-\lambda\tau)\;\Phi\left(\frac{R-\lambda\tau}{\sqrt{D\tau}}\right)\right\}$$

where $\phi(X)$ is the probability density function of a standard normal random variable X and $\Phi(X)$ will be the complementary cumulative distribution function of the same variable.

The expected unit years of backorder will be given by

$$B(R,T) = \frac{1}{T}\,[U(R,\tau+T) - U(R,T)] \tag{21}$$

where U(R, t) is given by

$$U(R,t) = \left[\frac{D^2-2\lambda^4 t^2}{4\lambda^3} + \left(\frac{D-2\lambda^2 t}{2\lambda^2}\right) R + \frac{R^2}{\lambda}\right] \Phi\left(\frac{R-\lambda t}{\sqrt{Dt}}\right) \tag{22}$$

$$+ \frac{1}{2}\left[D^{1/2} t^{3/2} - \frac{D^{3/2} t^{1/2}}{\lambda^2} - \frac{\sqrt{Dt}\, R}{\lambda}\right] \phi\left(\frac{R-\lambda t}{\sqrt{Dt}}\right)$$

$$- \frac{D^2}{4\lambda^3} e^{2\lambda R/D}\, \Phi\left(\frac{R+\lambda t}{\sqrt{Dt}}\right)$$

For this study the value of D will be assumed equal to λ. Substitution of D for λ in equations (20) and (22) gives

$$E(R,T) = \frac{1}{T}\left\{ \sqrt{\lambda(\tau+T)}\ \phi\left[\frac{R-\lambda(\tau+T)}{\sqrt{\lambda(\tau+T)}}\right] - [R-\lambda(\tau+T)]\, \Phi\left[\frac{R-\lambda(\tau+T)}{\sqrt{\lambda(\tau+T)}}\right]\right. \tag{23}$$

$$\left. - \sqrt{\lambda\tau}\, \phi\left(\frac{R-\lambda\tau}{\sqrt{\lambda\tau}}\right) + (R-\lambda\tau)\, \Phi\left(\frac{R-\lambda\tau}{\sqrt{\lambda\tau}}\right)\right\}$$

and

$$U(R,t) = (1 - 2\lambda^2 t^2 + 2R(1-2\lambda t) + 2R^2)\, \Phi\left(\frac{R-\lambda t}{\sqrt{\lambda t}}\right) \tag{24}$$

$$+ \frac{1}{2}\left(\frac{\lambda t - {}^{3/2} t^{1/2}(1+R)}{\lambda^{1/2}}\right) \phi\left(\frac{R-\lambda t}{\sqrt{\lambda t}}\right) - \frac{1}{4\lambda} e^{2R}\, \Phi\left(\frac{R+\lambda t}{\sqrt{\lambda t}}\right)$$

The expected unit years of storage will be given by

$$D(R,T) = R - \frac{\lambda T}{2} - \mu + B(R,T) \tag{25}$$

where μ is the expected lead time demand.

The cost of ordering will be A/T because there will be 1/T orders placed per year. Also the review cost will be J/T for the same reason. Because an order is placed at every review these two costs can be combined into a single cost L. So L will be A+J, and the total annual cost for ordering and reviews will be L/T. Combining all the terms for the total annual cost gives

$$K = \frac{L}{T} + IC\left(R-\mu - \frac{\lambda T}{2}\right) + \pi E(R,T) + (IC+\hat{\pi})\ B(R,T) \tag{26}$$

To solve this model the method followed was to determine the optimum values of R for various values of T. The total cost was computed for each combination of R and T, and then the optimal policy was the one with the minimum total cost.

CHAPTER III

SOLUTION PROCEDURE

Two Fortran V computer programs were written to perform the necessary calculations to do the comparison between the deterministic inventory models and the two stochastic models. For convenience the program for the comparison between the deterministic model and the (Q,r) model will be called the "Q Program" and the one for the comparison between the deterministic model and the (R,T) model will be called the "R Program." Each of these programs is divided into three parts. The first part computes the deterministic policy, the second part computes the stochastic policy according to the procedure outlined in the previous chapter, and the last one performs the comparison between the two policies.

To measure the effectiveness of the deterministic model in approximating the stochastic system, a variable called Percent Deviation, PD, was defined. This variable gives the percent cost increase that is incurred when the deterministic policy is used to operate the stochastic system instead of the stochastic policy. It is defined as

$$PD = \frac{TCS2 - TCS1}{TCS1} \qquad (27)$$

where TCS1 is the expected annual total cost if the system is operated using the stochastic policy and TCS2 is the expected total cost if the system is operated using the deterministic policy.

The value of this variable was determined for all the combinations of the parameters at the extreme values of their ranges. This was done to indicate which of the parameters of the systems and their two way interactions were important in the analysis. Their corresponding sums of square terms were determined and the terms that made the largest contribution to the total sums of squares were chosen for further study. Additional runs were made for the important terms to study the sensitivity of the PD to changes in their values.

When a deterministic policy is being used it is not necessary to use any special type of system to control the inventory levels as is required when a stochastic policy is used. The deterministic policy orders a certain amount at fixed points in time, because it assumes that the inventory levels are known exactly at every point of time. When a stochastic policy is used some type of control system is required to make available to the decision maker the state of the system. This in turn creates the additional cost of operating the system which is not incurred with the deterministic policy. Of course, no matter what type of policy is used physical counts of the inventories will have to be done occasionally to up date the records and eliminate errors due to spoilage, breakage, and other losses of inventory items. This additional cost is the difference in operational cost incurred when a stochastic model is used to represent the stochastic system. A sample run was made with an arbitrary value of this operating cost to study the sensitivity of the PD to this cost.

A detailed solution procedure for each one of the two comparisons made will now be given.

Q Program

This program performs the necessary calculations to do the comparison between the deterministic model with infinite rate of resupply where backorders are allowed with the stochastic lot size reorder point model with infinite rate of resupply. The first segment of the program reads the parameters of each item and finds the optimal Wilson lot size, QW, and total cost for this policy, TCW. It then determines which case of the possible combinations of the values of π and $\hat{\pi}$ each item has so that the appropriate solution procedure to obtain the optimal policy can be used. Next the optimal deterministic policy is determined for each item. The flow chart for this segment is given in Figure 4.

The second part of this program finds the optimal stochastic policy for each item, subject to the accuracy of the numerical procedures used. Here the iterative procedure recommended by Hadley and Whitin (12) is followed. To start the search the value of Q is made equal to QW, which is available from the first segment of the program. This value is substituted into equation (19) to find a value of r; the following search procedure is used. The value of r is set equal to the deterministic lead time demand. The value of the right hand side of equation (19) is then computed and compared with the left hand side which is independent of r. For this particular value of r the right hand side which will be called B, will always be greater than the left hand side which will be called F. The value of B will decrease as r increases and F will remain constant. Then r is increased by increments of a unit until B becomes less than F. The value of r for which the change occurred is reduced by

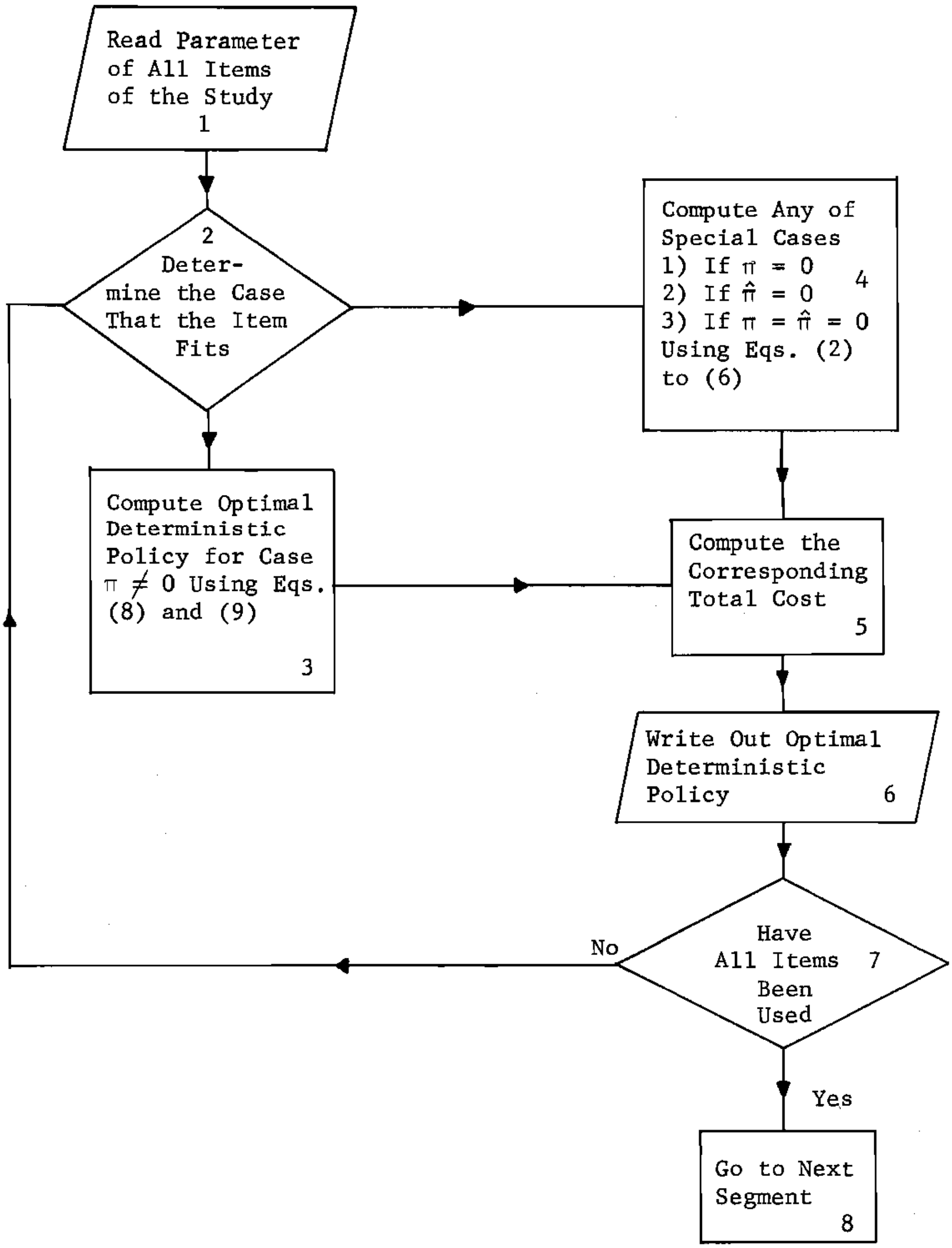


Figure 4. Flow Chart of First Segment of Q Program

one increment and the increment is also reduced and the cycle is repeated until the increments on r are reduced to 0.01 and B again becomes less than F. This value of r is then substituted into equation (18) and a new value of Q is obtained. This value is compared with the previous value of Q and if they differ by less than 0.01 the search is stopped. These last values of Q and r will be approximately the optimum stochastic policy. Otherwise, the program uses the last value of Q to restart the cycle. Finally it computes the total stochastic annual cost. The flow chart for this segment is given in Figure 5.

The last segment of the program computes the value of the PD for each one of the items using the optimal stochastic and deterministic policies found in the previous segments. Finally it will add the additional cost of operating the system to the stochastic cost and find a new stochastic cost, TCS3. It then computes the new value of PD as

$$PD = \frac{TCS2 - TCS3}{TCS3} \tag{28}$$

The flow chart for this segment is shown in Figure 6.

A listing of the complete program and sample output for 64 items is given in Appendices II and IV, respectively.

Data Selection

The parameters needed to determine the optimal inventory policies for each item are average demand per year, ordering cost, unit cost, carrying charge, fixed backorder cost, variable backorder cost, lead time, and the standard deviation of the demand distribution during lead time. The ranges for these parameters were chosen to include the most commonly

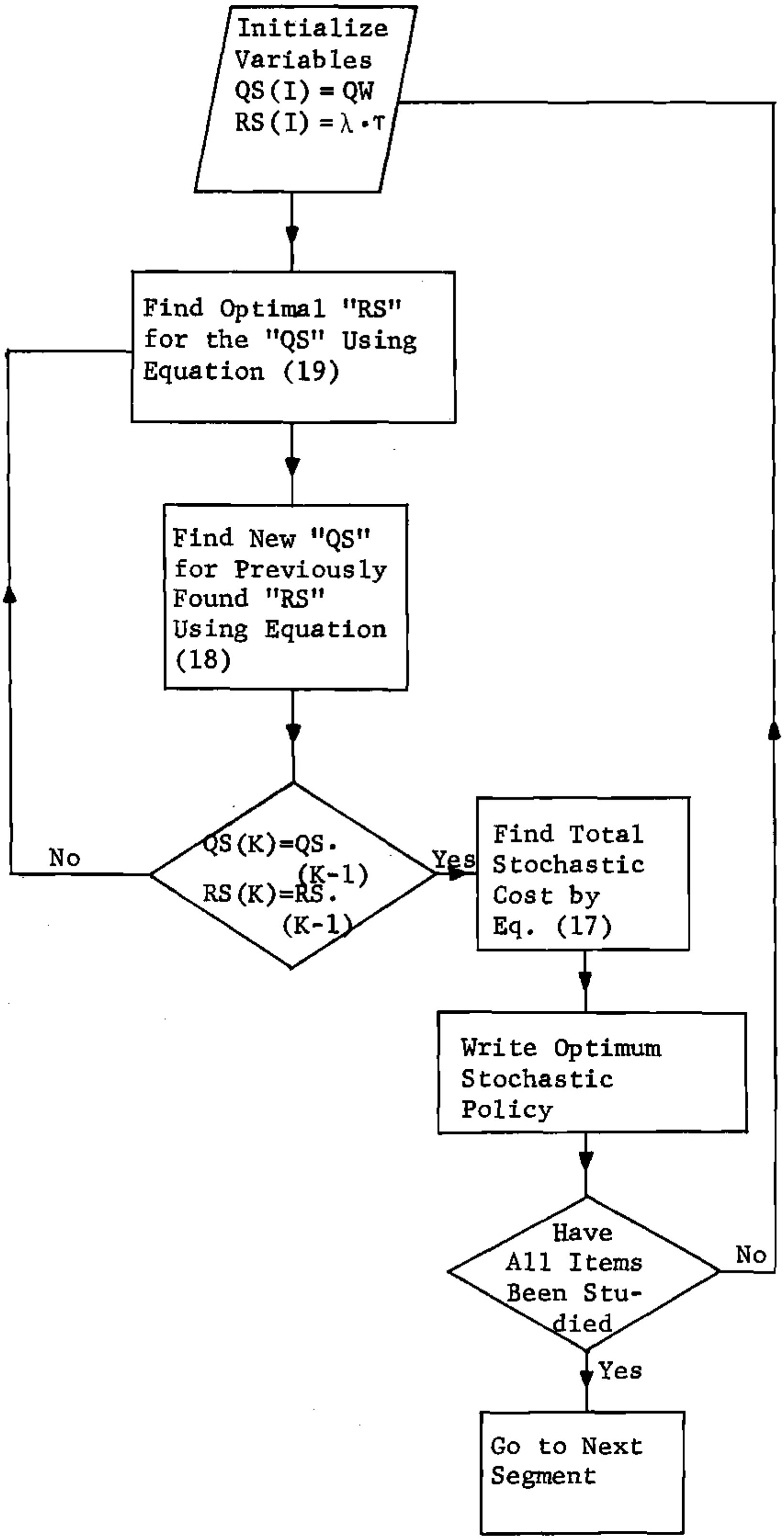


Figure 5. Flow Chart of Second Segment of Q Program

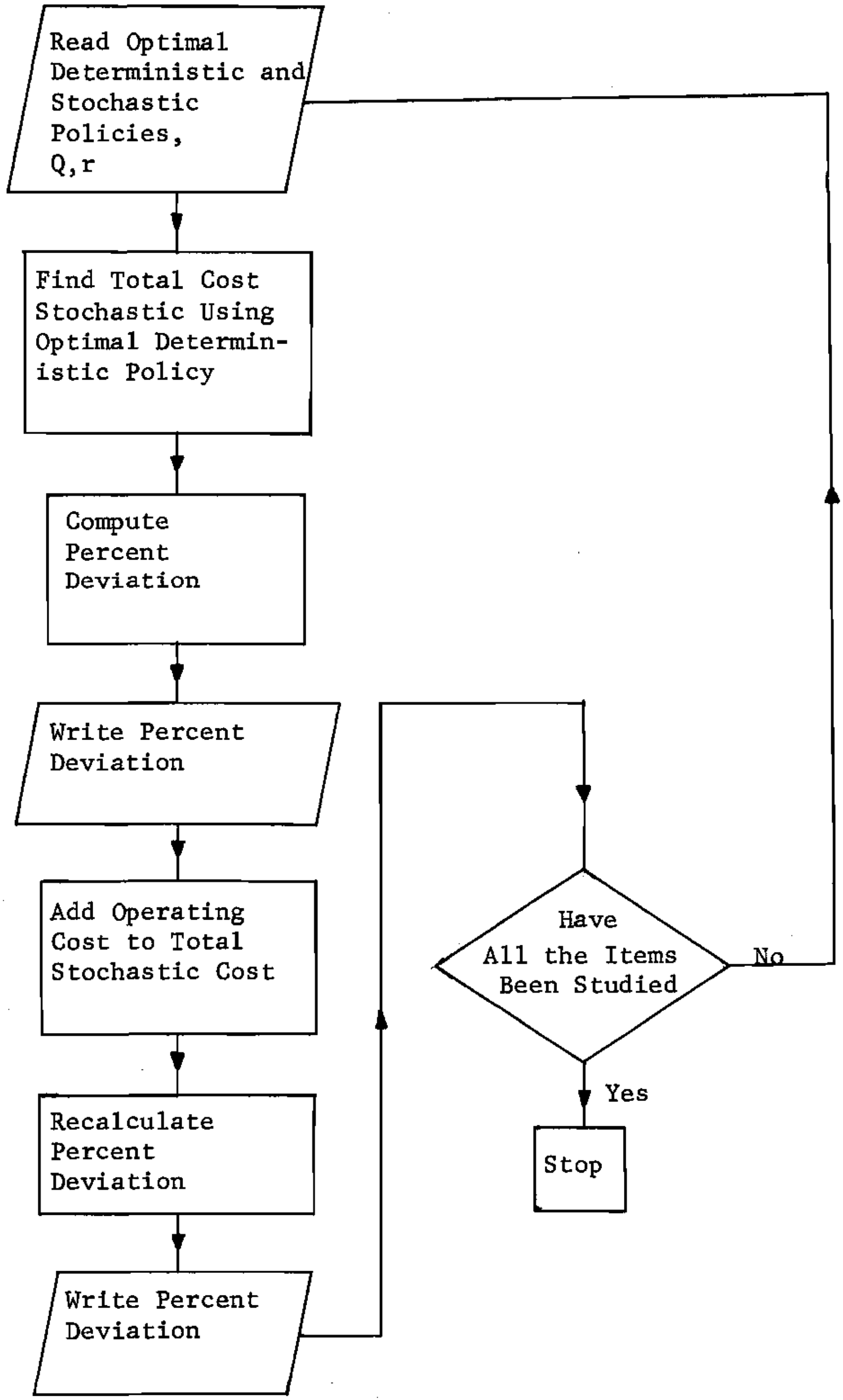


Figure 6. Flow Chart of Third Segment of Q Program

used values found in inventory literature. No attempt was made to represent any particular situation. To find these ranges for the parameters several books, articles, and practical studies were analyzed such as (5,12,15,22,32). The only parameter which all the authors agreed on was the value of the carrying charge. All agree that it should have a value in the neighborhood of 0.20 dollar per dollar year invested in inventory; therefore, this value was chosen for the study. The value of the standard deviation of the demand distribution was left to be varied at will so no range was determined for it. The ranges chosen for the study for the rest of the parameters are shown in Table 1.

Table 1. Ranges of Parameters

Parameter	λ	A	C	π	$\hat{\pi}$	τ
Dimensions	unit/year	$/order	$/unit	$/bckord	$/bckord-yr	year
High Level	3500	450	300	1.0	1000	0.1
Low Level	400	10	8	0.1	350	0.03

Some slight modifications were made on ranges found in the literature so that unrealistic situations were avoided and the results and methodology could be shown more clearly. An additional comparison, after adding a certain operating cost to the optimal total stochastic, was performed. Due to the fact that this cost could vary from several dollars to several thousand dollars depending on the degree of sophistication of the operating system used, one arbitrary value was chosen as an example.

For it, a comparison was made to determine the sensitivity of the PD to the additional operating cost. The value chosen was $100 per item year, considering that the total additional operating cost is divided evenly between all the items of the inventory system. The results of this analysis are shown in Appendix VI.

Sensitivity Analysis

An analysis of the sensitivity of the PD to changes in values of the seven parameters considered in this study was performed. First, all of the possible combinations of the seven factors at their high and low levels were determined. Then a run of the Q program using the 128 data set was performed to find the values of the PD for each combination. Using these values the sums of the square terms were determined, as they are calculated in an analysis of variance, for the seven parameters and for all their two way interactions. Higher order interactions were not considered. From these values the terms that contributed the most to the sums of the square terms were chosen so that their effect on the PD could be determined by further runs of the Q program.

Graph Construction

For each one of the terms that appears to be important from the previous analysis, an additional run with the Q program was made to study the variation of the PD with changes in these terms. To do so, all the parameters except the one for the effect being investigated were held constant. The effect was investigated for values of the parameters within the range previously determined. Also for the parameters being held constant, three levels were chosen. That is, the study of the effect for every parameter was performed for three levels of the rest of the parameters, all at high level, all at the midpoints of their corresponding ranges, and all at low level. For each one of these combinations, the PD was found for certain values of the parameter being studied and a graph was made of PD versus the parameter. These graphs are presented in Chapter IV where a discussion of the results is presented.

The parameters that were found not to be important were held constant at an intermediate level for all the calculations performed.

R Program

This program compares the deterministic inventory model with infinite resupply rate where backorders are allowed with the fixed cycle periodic review model with infinite rate of resupply. The first segment of this program is exactly the same as the first segment of the Q program.

The second part finds the stochastic policy for each item. A double search on the two control variables, the "order up to" level, R, and the fixed interview time, T, was performed to find the optimal stochastic policy. This double search procedure consists of first doing a search for the value of R for a given T and then from the sequence of values of T finding the least costly one. The search procedure starts at T being equal to one day and finds the optimum R for that value of T. The value of the total cost is used as a control to find both optimum values. As T increases, the value of the total cost will first decrease and then increase. Based on this fact, T is incremented by increments of ten days until the total cost starts increasing. Then the value of T is decreased by two increments, the increment is reduced to one day and the process is repeated until the total cost starts increasing again. At this point the value of T that is taken as optimum will be the one immediately previous to the switch of direction of the total cost. The same criterion is used to determine the optimum value of R for each value of T, but the increments in this case will be reduced as far as 0.1 unit. Also, the search on R starts always with a value of R equal to the deterministic lead time demand which is available from the first segment of the program. After the stochastic policy is found, the total stochastic annual cost is calculated. The flow chart for this segment is given in Figure 7.

The third segment of the program is equivalent to the third segment of the Q program. A flow chart of this part is given in Figure 8. Also a listing and output of the program are given in Appendices III and V.

Data Selection

The data used for this program were exactly the same data used in the Q program. However, an additional parameter must be determined. This parameter is the review cost. In order to make the comparison to determine which of the three models (the (Q,r) model, the (R,T) model, and the deterministic model) is the most economical one to implement, this review cost must either be zero or must be included in the ordering cost. If this cost was different than zero then the (R,T) model would always give a higher total annual cost than the (Q,r) model. Therefore, for the cases in which the deterministic model would be more economical than the (Q,r) model, no comparison with the (R,T) model would be needed. So the review cost was considered to be included in the ordering cost and no additional information was required to run the program.

Due to several problems encountered with the (R,T) model, no addi-

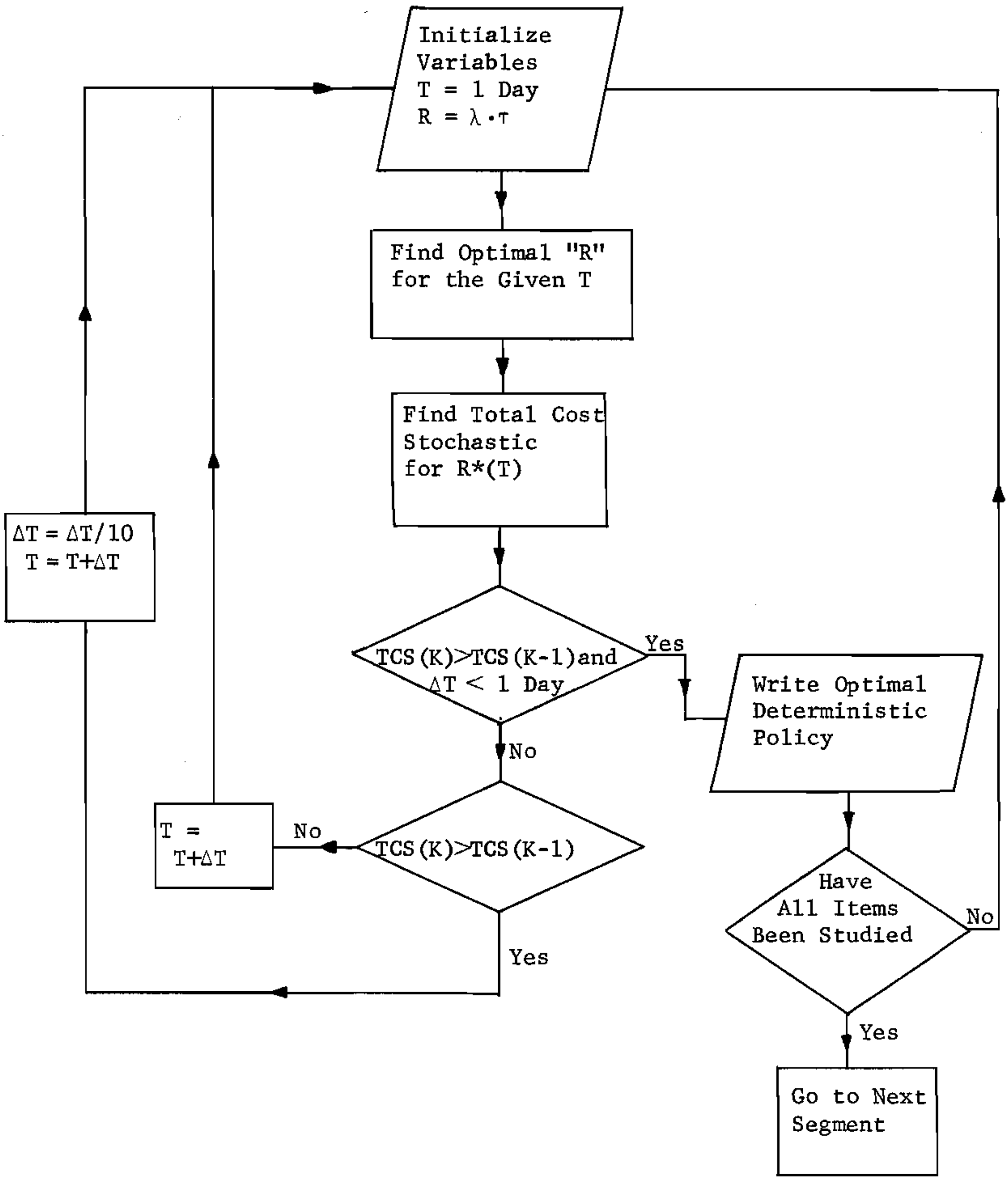


Figure 7. Flow Chart of Second Segment of R Program

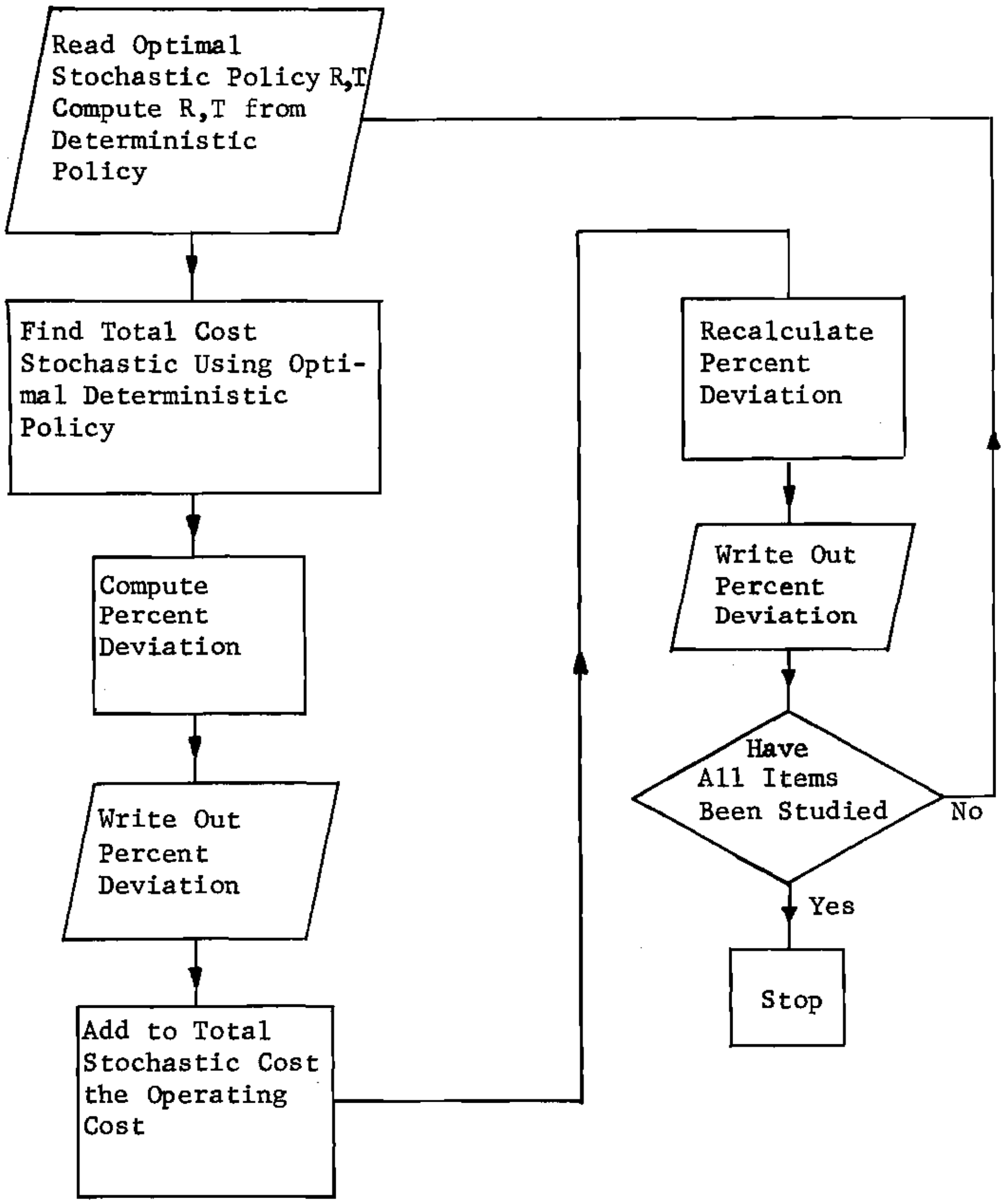


Figure 8. Flow Chart of Third Segment of R Program

tional studies were made to compare between the (R,T) model and the deterministic model. These problems will be discussed in Chapter IV where a discussion of the results will be presented.

CHAPTER IV

DISCUSSION OF THE RESULTS

Results for Q Program

The discussion of the results of the (Q,r) model will be presented first. The sensitivity analysis performed on the PD obtained from this model shows that the lead time, τ, the fixed backorder cost, π, and all their second order interactions contribute less than two percent of the total sum of the squares and, thus, were not considered for further investigation. Also, of the remaining two way interactions, only the $\sigma\hat{\pi}$, σC, and $C\hat{\pi}$ interactions were deemed to be unimportant because each contributes less than two percent to the total sum of the squares. Only the effect of the remaining parameters and two way interactions was considered further. The results of the sensitivity analysis are shown in Table 2.

The effect that each of the relevant parameters had on the PD was studied for three different levels of the rest of the parameters. Curves were drawn for each of these cases holding the parameters that were not being analyzed constant at their maximum, midpoint, and minimum values of their ranges used for the study. These curves were labeled on the graphs as low, medium, and high levels, respectively. The parameters that were not relevant were held constant at the midpoint of their ranges.

Table 2. Sums of Square Terms for Q Program

Parameters and Interactions		Sums of Square Terms	Percent of Total Sums of Squares
σ		29.12	6.14
λ		55.08	11.62
A		101.67	21.45
C		22.54	4.76
π		.11	.02
$\hat{\pi}$		25.29	5.33
τ		.00	.00
$\sigma\lambda$		13.05	2.75
σA		25.39	5.36
σC		6.59	1.39
$\sigma\pi$		.01	.00
$\sigma\hat{\pi}$		6.04	1.27
$\sigma\tau$		.00	.00
λA		48.52	10.24
λC		11.66	2.46
$\lambda\pi$		.01	.00
$\lambda\hat{\pi}$		12.40	2.61
$\lambda\tau$		.00	.00
AC		20.32	4.29
$A\pi$		.11	.02
$A\hat{\pi}$		22.08	4.66
$A\tau$		.00	.00
$C\pi$		.05	.01
$C\hat{\pi}$		5.02	1.06
$C\tau$		.00	.00
$\pi\hat{\pi}$		.00	.00
$\pi\tau$		.00	.00
$\hat{\pi}\tau$		.00	.00
Rest of terms		68.86	14.53
	Total	473.93	100.00

Main Effects

The parameters of the system that turned out to be important, as shown in Table 2, were the ordering cost, A, the average number of demands per year, λ, the standard deviation of the demands during lead time, σ, the variable backorder cost, $\hat{\pi}$, and the unit cost, C. The ordering cost, A, was the most important with 21.5 percent of the total sums of squares, then λ with 11.6 percent, σ with 6.1 percent, $\hat{\pi}$ with 5.3 percent, and C with 4.75 percent.

The graph of percent deviation, PD, versus A is shown in Figure 9. As can be seen in this graph, if the value of A increases, the value of PD decreases. For low values of A the values of the PD increase very rapidly. This behavior might be due to the fact that, for very low values of A, the model will determine small reorder quantities, Q, and reorder points, r. This is because it would be cheaper to order very frequently instead of holding items in stock. These policies will be more sensitive to small differences in the values of Q and r than policies where Q and r have higher values, for the same absolute differences. This sensitivity is reflected in the change of the relationship between the optimal holding and shortage costs. When the deterministic policy is used to operate the system instead of the optimal stochastic policy, both Q and the reorder point r will be smaller than the optimal stochastic ones. This variation will make the system incur a higher number of backorders than would be optimal. Thus, in general the backorder costs will be high. Therefore, the savings obtained in the holding cost due to the reduction of the number of items in inventory will be nullified by the increase in shortage cost. As the rest of the parameters decrese, $\hat{\pi}$ will decrease and so lower

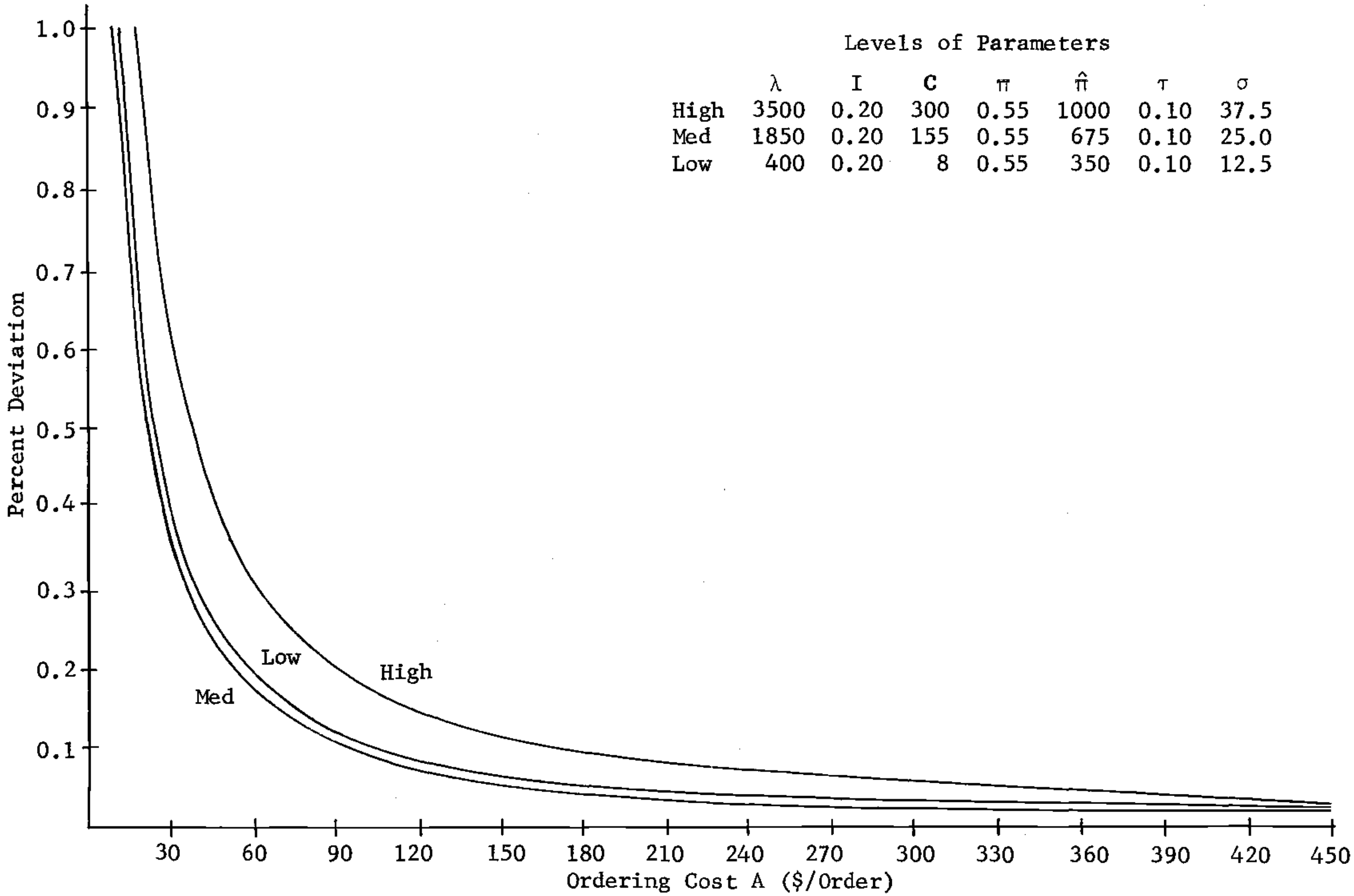


Figure 9. Graph of Percent Deviation vs. A (Q,r) Model

deviations will be obtained as the rest of the parameters decrease.

It can be observed also that, according to the previous discussion, the expected relative position of the curves for the low level of the rest of the parameters is changed. This is due to the fact that, for low values of λ and C, very high values of the PD are obtained, as will be shown later. Therefore, the combined effect of these two parameters would counterbalance the reduction in PD due to the reduction in the values of $\hat{\pi}$ which in turn will make the logical values of the PD for the low level higher than they should be.

Also, as A gets larger, the difference between the curves reduces. This is because as A gets larger the effect of the absolute level of the rest of the parameters becomes less significant. As A rises above \$150 per order for the ranges of parameters used in the study, the value of the PD is under 10 percent; also, as A drops below \$30 per order, the values of the PD will never get below 35 percent.

The next parameter in importance is λ. The graph of PD versus λ is shown in Figure 10. As λ increases the value of PD decreases regardless of the values of the other parameters. As λ decreases the value of the PD increases very rapidly especially for the low level curve. This is because, as λ increases for a fixed value of σ, the coefficient of variation, defined as

$$CV = \frac{\sigma}{\mu} \tag{29}$$

where μ is the expected lead time demand, will decrease. This will make the distribution of the number of the demands during lead time have less significant variation, and the points will be centered relatively more closely around the mean. Because for the study the deterministic lead

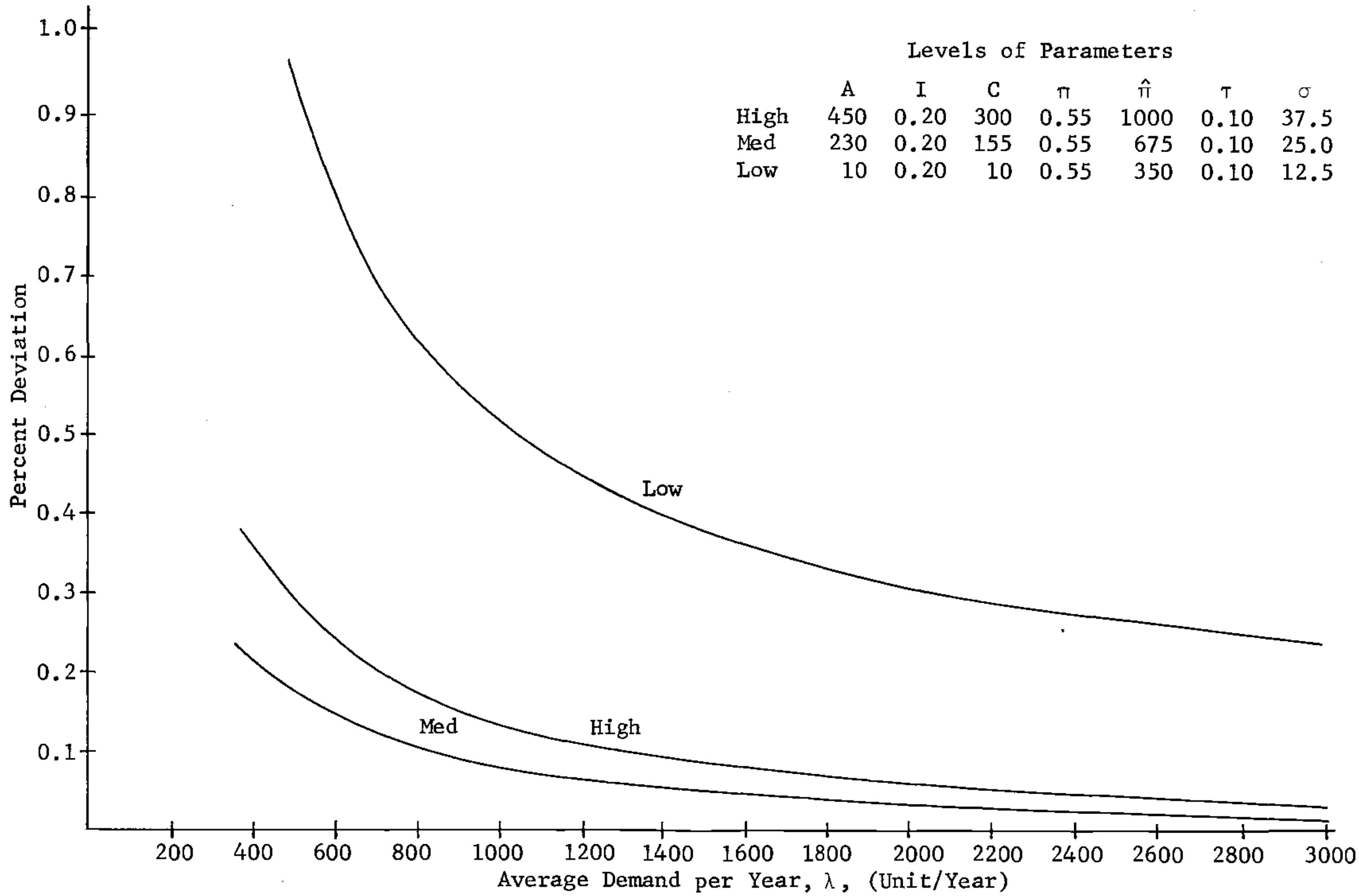


Figure 10. Graph of Percent Deviation vs. λ (Q,r) Model

time demand used is equal to the expected lead time demand for the stochastic case, the two policies will be closer together and less PD will be observed. When σ is at higher levels the value of the CV will be larger and higher values of PD will be obtained. Therefore, for higher levels of the rest of the parameters, higher values of PD will be obtained due to higher shortages costs.

As the value of the other parameters goes from high to medium levels, the reduction of A from \$450 to \$230 per order will produce a smaller increase in the PD than the decrease produced by σ when it drops from 37.5 to 25 units. Therefore, for this change of levels, the predominant effect is the one due to σ and lower values for the PD will be obtained for the medium level curve than for the higher level curve. The change in the relative position of the low level curve is due to the fact that, for that level, the deterministic policy turns out to be the Wilson policy where no backorders are allowed. Therefore, for this case the deterministic and stochastic policies will differ more than if the deterministic policy compared allowed backorders. This case corresponds to a comparison of two models one of which (the deterministic) is different from the one used in the high and medium curves. This is why bigger values than would be expected for the PD are obtained. As the levels change from medium to low, A goes from \$230 to \$10 per order and σ goes from 25 to 12.5 units. Here the decrease obtained for PD from the reduction of σ is small compared with the increase obtained from the reduction of A. For this change in levels, the predominant effect is the A effect, and higher deviations are obtained. This effect combined with those previously discussed produces the high values of PD obtained for the low level curve.

As the value of λ increases, the difference between the curves reduces. This indicates that, as λ gets larger, the effect of the absolute values of the rest of the parameters becomes less important.

For the ranges of parameters used, it can be seen that, for the high and medium levels, as λ gets greater than 700 units per year the value of the PD is under 20 percent and if it is over 1200 units per year the PD is under 10 percent. No matter what the value of λ is, if the rest of the parameters are at low levels the PD will never get under 20 percent for the ranges used.

Next in importance is the standard deviation of the demands during lead time, σ. The graph of PD versus σ is shown in Figure 11. Examining the graph it can be seen that, as the value of σ increases, regardless of the value of the rest of the parameters, the PD will increase. Also, as the level of the parameters decreases, the value of the PD will also increase. This is because, as the value of σ increases, the value of the CV will get larger and, therefore, the variation of the demand will become larger. The values of the stochastic policy will be much greater than the values for the deterministic one and this will increase the value of the PD. Also, as the rest of the parameters decrease, the value of the CV will increase because μ will get smaller as λ does. This again produces higher PD as the levels of the parameters not being studied decrease.

The huge values of the PD for the low curve are due to the same causes presented for the previous case. That is, that the deterministic policy used in the comparison is the Wilson policy and that the relative magnitude in change of the PD is due to changes in level of the other parameters.

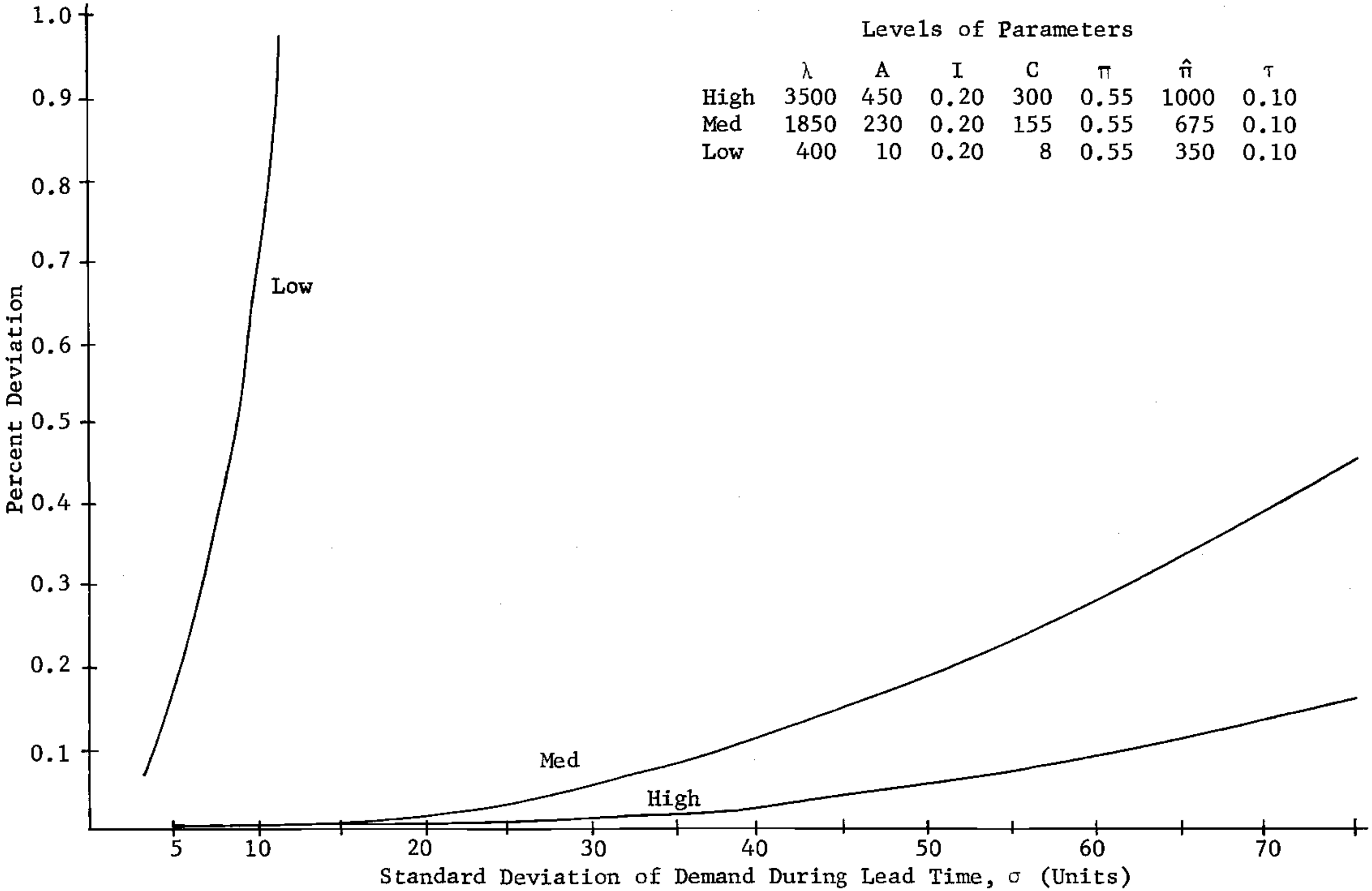


Figure 11. Graph of Percent Deviation vs. σ (Q,r) Model

Here also as the value of σ increases the curves for the different levels of the other parameters diverge. This means that the effect of the absolute values of the parameters becomes more relevant as σ increases.

For the ranges used, if the levels of the parameters are equal or higher than the medium level used, the PD will be less than 20 percent if σ is less than 50 units and will be less than 10 percent if σ is less than 35 units.

The next parameter analyzed is the variable backorder cost, $\hat{\pi}$. The graph of PD versus $\hat{\pi}$ is shown in Figure 12. It can be seen that, as the values of $\hat{\pi}$ increase, the values of the PD also increase. Also, as the level of the rest of the parameters decreases, the values of PD increase for a given value of $\hat{\pi}$. The values for the low level differ so much from the ones corresponding to the other two levels because of the same reasons explained in the λ and σ effects for the same situation.

As the value of $\hat{\pi}$ increases, the effect of the absolute value of the other parameters also becomes more relevant, and the curves for the different levels diverge as $\hat{\pi}$ increases.

It may also be noticed that the effect of $\hat{\pi}$ is linear for a given level of the rest of the parameters and that changes in its values do not affect greatly the amount of percent deviation incurred. This is an advantage because, in general, the value of $\hat{\pi}$ is very difficult to determine precisely. Thus, errors in its determination will not greatly affect the final value of the PD for any case.

As long as the rest of the parameters remain at levels higher than the medium level, the value of PD will always be less than seven percent no matter what value $\hat{\pi}$ has. For value of $\hat{\pi}$ less than \$350 per unit year

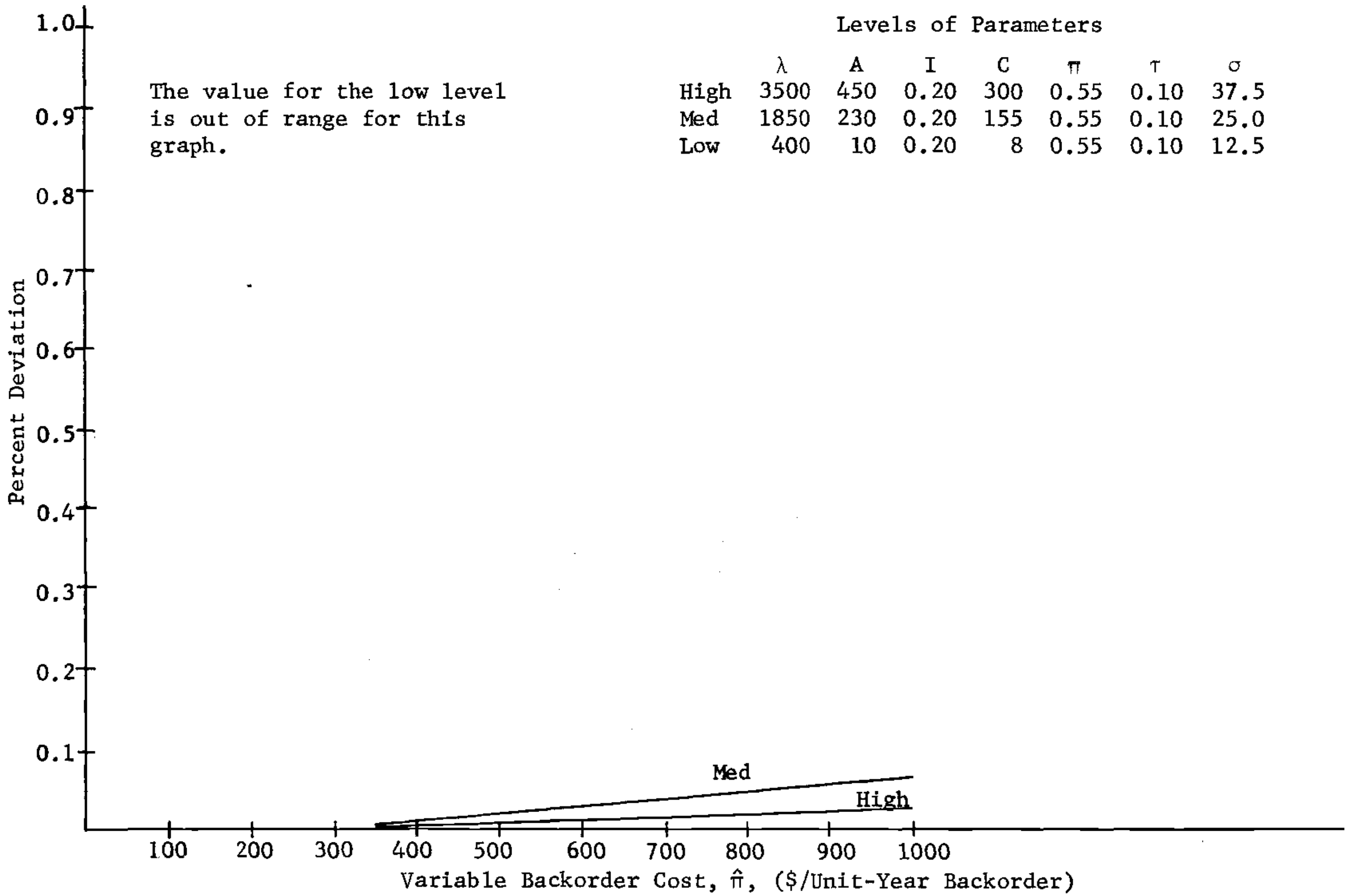


Figure 12. Graph of Percent Deviation vs. $\hat{\pi}$ (Q,r) Model

of backorder, the effect of $\hat{\pi}$ is negligible on the values of PD.

The last important parameter as determined by the sensitivity analysis is the unit cost, C. The graph of PD versus C is shown in Figure 13. If the values of the rest of the parameters remain above the medium level, the value of C has almost no effect on the value of PD. If the values are at low level, then the effect of C is more relevant. As C increases, regardless of the values of the other parameters, the PD will decrease. Also, as the level of the other parameters decreases, the value of PD increases for a given value of C.

The big difference in the PD values for the low level might be due to the combined effect of the other parameters being at low level.

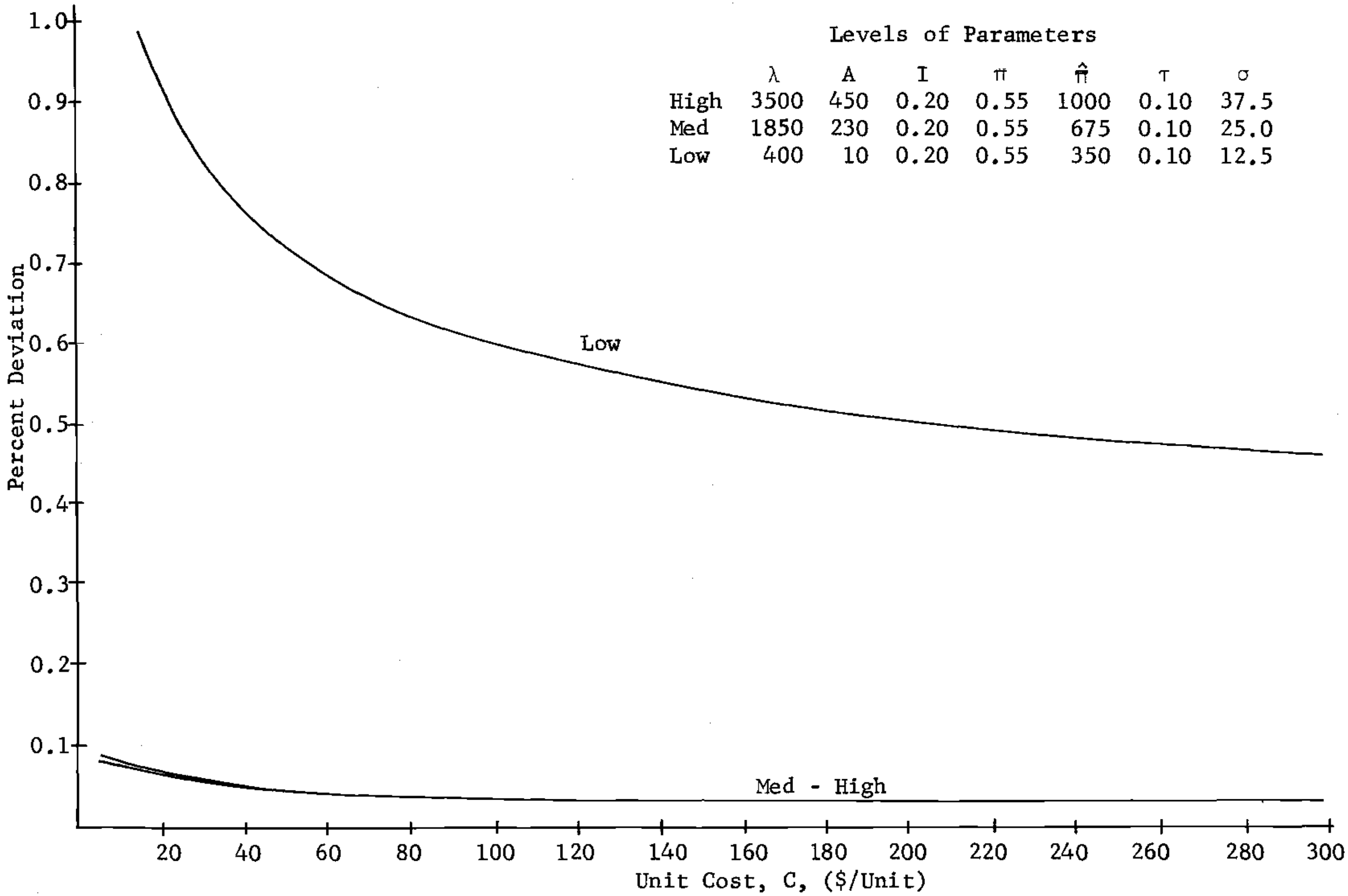
Also, as C increases, the difference between the curves gets smaller. This would indicate that the effect of the values of the other parameters decreases as C increases. For the ranges chosen, if the parameters are at low level, the PD will never be less than 45 percent and if they are at medium and low levels the value of PD will never be greater than 10 percent regardless of the value of C.

In general it can be concluded that the ideal situation for which the stochastic system can be operated using the deterministic policy will be the one with the following characteristics.

1. The ordering cost, A, the average number of demands per year, λ, and the unit cost, C, must be at relatively high values.

2. The cost of backorders, $\hat{\pi}$, and the standard deviation of demands during lead time, σ, must be at small values.

Of course this ideal situation might not be in general encountered, but as long as certain parameters counterbalance the effects of other



Levels of Parameters

	λ	A	I	π	$\hat{\pi}$	τ	σ
High	3500	450	0.20	0.55	1000	0.10	37.5
Med	1850	230	0.20	0.55	675	0.10	25.0
Low	400	10	0.20	0.55	350	0.10	12.5

Figure 13. Graph of Percent Deviation vs. C (Q,r) Model

parameters, the deterministic policy might still be applicable, i.e., if σ is at a high level but A is also at a very high level, their effect will counterbalance.

In general, as long as low values of the PD are obtained, the deterministic policy due to its simplicity in implementing and operating will be the most logical one to use. Up to now no special consideration has been given to the additional cost for operating the system when the stochastic policy is used. This additional cost will favor even more the application of the deterministic policy to solve stochastic systems.

In Table 3 a summary of the effect that the parameters have on the values of the PD is presented. In each, all the effect that the parameter has on the PD as the parameter increases for the different levels of the rest of the parameters, is shown. The parameters are presented in order of importance.

As can be seen, the parameters for which the PD was more sensitive to changes in their values turned out to be in the same order of importance as predicted by the analysis of the sums of square terms.

Interaction Terms

From the sensitivity analysis it was determined that some of the two way interactions were important in terms of their contribution to the total of squares. The most important one was the $A\lambda$ interaction which was 10.24 percent of the total sum of the squares, then σA with 5.36 percent, $A\hat{\pi}$ with 4.66 percent, AC with 4.30 percent, $\sigma\lambda$ with 2.75 percent, $\lambda\hat{\pi}$ with 2.61 percent, and λC with 2.46 percent. The rest of the interactions contributed less than 1.5 percent each and were not investigated in detail. The 99 higher order interactions were not considered because,

Table 3. Summary of Analysis of Main Effects

Main Effects \ Other Parameters	LOW	MEDIUM	HIGH	REMARKS
A	Decreases at high rate for low values of A	Decreases at high rate for low values of A	Decreases at high rate for low values of A	At high values of A, almost same values of PD for all levels of rest of parameters
λ	Decreases faster for low values of λ	Decreases faster for low values of λ	Decreases faster for low values of λ	At high values of λ for high and medium levels of rest of parameters, almost same values of PD
σ	Increases at very high rate for all values of σ	Increases moderate rate for all values of σ	Increases low rate for all values of σ	As σ gets larger, the rate of increase gets larger for all levels of rest of parameters
$\hat{\pi}$	Increases at low rate	Increases at low rate	Increases at low rate	The relationship with PD is linear
C	Decreases at very low rate	Decreases at very low rate	Decreases at very low rate	For high and medium levels of rest of parameters, the effect of C becomes negligible as C increases

jointly, they contributed only 14.53 percent of the total sum of the square terms.

The graph of the PD versus the ordering cost, A, for different values of the number of demands per year, λ, is presented in Figure 14. It can be seen that, as A and/or λ increase the value of the PD decreases, no matter what the levels of the other parameters are. For very small values of A, very high values of the PD are always obtained. These behaviors can be explained by the same arguments presented for the main effects.

Also, as the level of the rest of the parameters decreases for given values of λ and A, the values of the PD decrease. This is due to the fact that, as the σ and $\hat{\pi}$ values decrease the PD will also decrease and their combined effect will nullify the effect that as C decreases the PD will increase.

The fact that an interaction between A and λ exists can be observed since for a given level of the other parameters the curves for different values of λ are not parallel. The difference between curves for different values of λ changes as A varies. This interaction remains approximately the same for all ranges. Only when A gets very big does the interaction get less significant. This can be seen in the graph by the fact that the curves tend to get more parallel for high values of A.

Moreover, as A increases, all the curves get closer together. This means that the effect of the other parameters is counterbalanced by the effect of A. For the ranges chosen for the study, if A is greater than \$170 per order and λ is greater than 1800 units per year, the value of PD will be less than 10 percent. But, if A is less than \$10 per order,

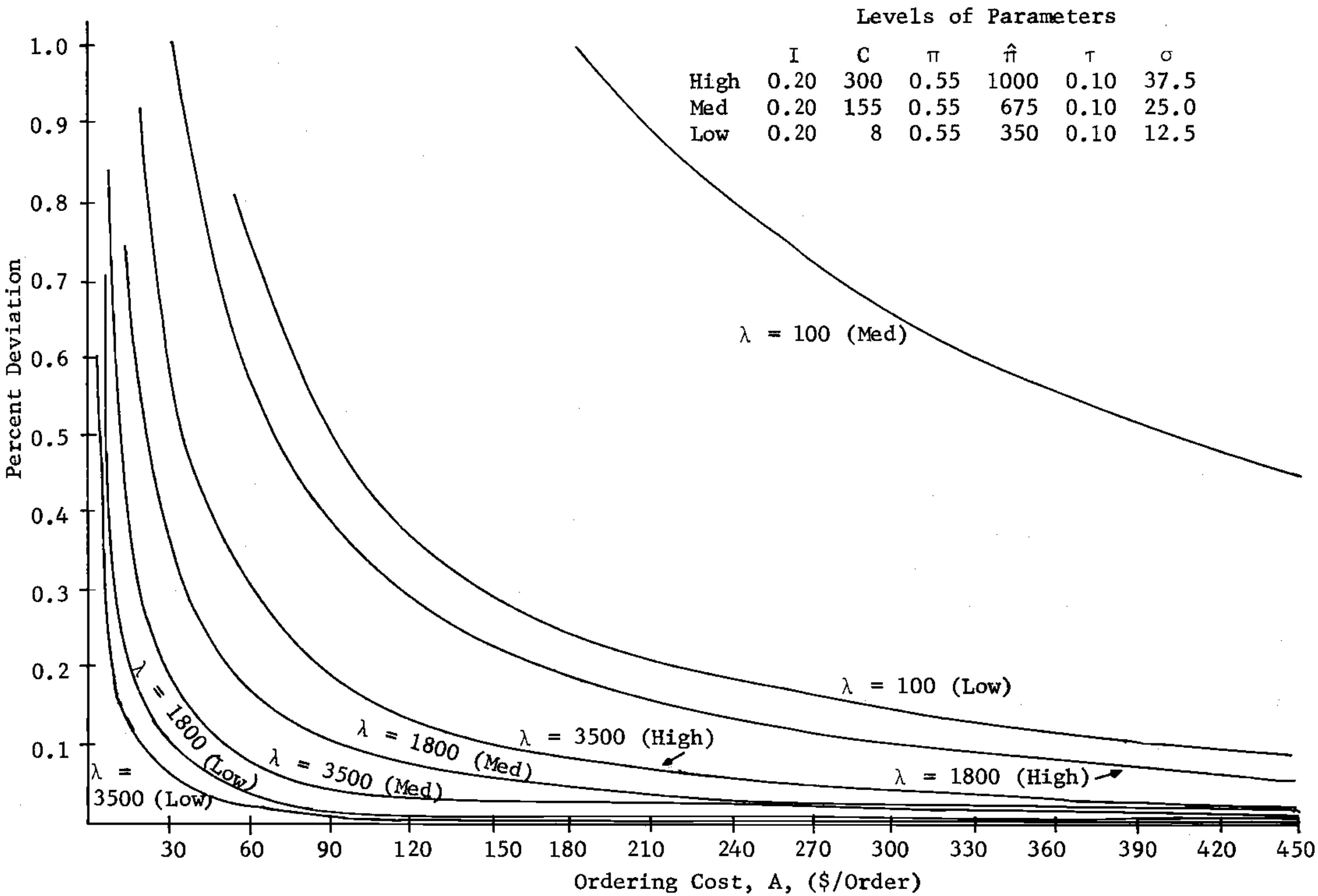


Figure 14. Graph of Percent Deviation vs. λA (Q,r) Model

the value of the PD will be greater than 20 percent for all values of λ under 3500 units per year.

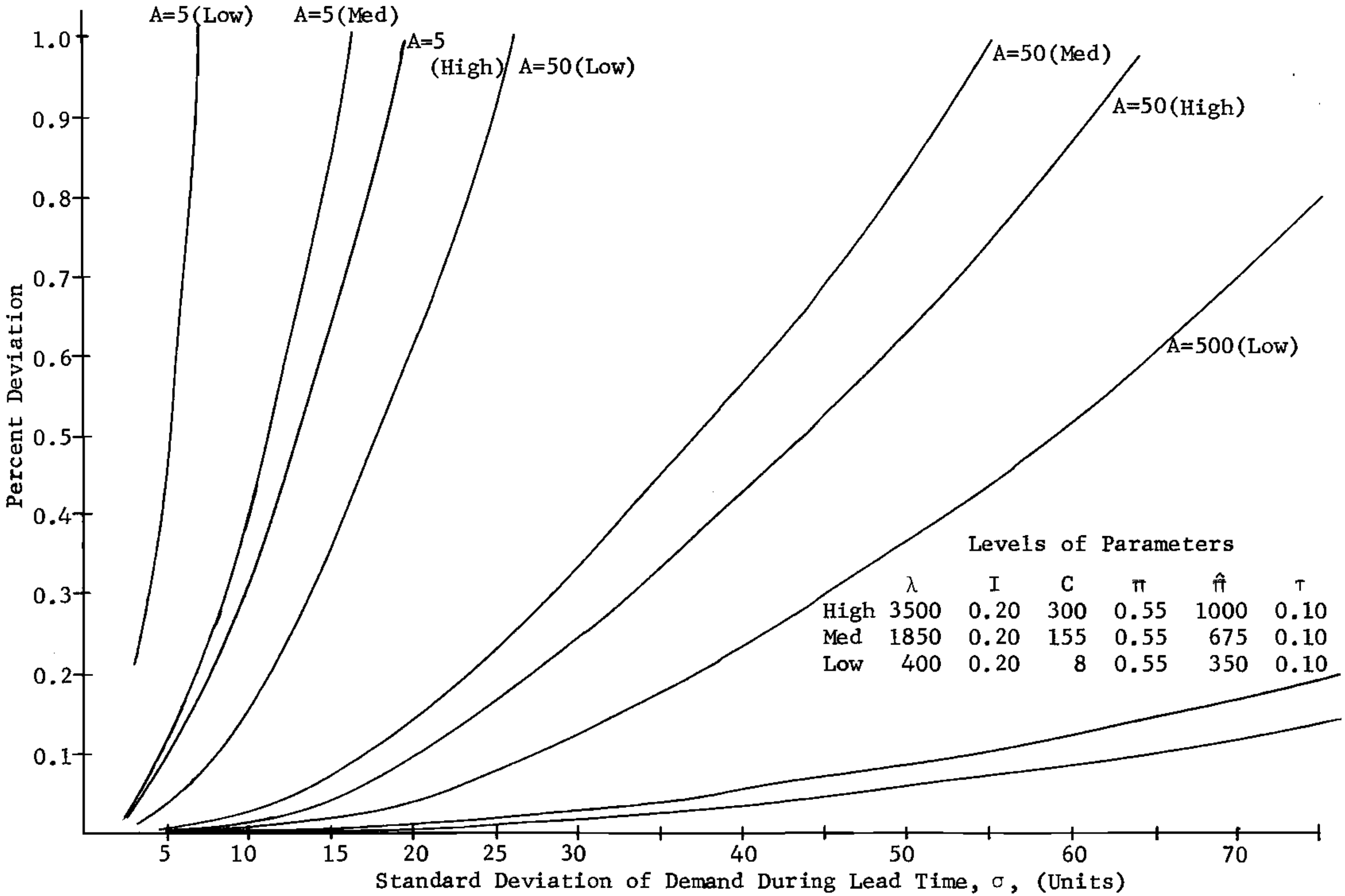
The next interaction considered is σA. The graph of PD versus the standard deviation of demand during lead time, σ, for different levels of A can be found in Figure 15. As σ increases and A decreases the values of PD increase. Likewise, as the value of the rest of the parameters decreases the values of the PD increase. This behavior can be explained by the same arguments as used in the A and σ effects. The very high deviations obtained for the cases where A is low are due to several reasons. Besides the effect of A being at low level which by itself will give very high deviations, λ is at low level, so higher values of CV will be obtained which in turn gives higher values of the PD for every value of σ. Also, for this case, the deterministic policy found is the one where no backorders are allowed. Therefore, the combined effect of all these conditions will give the high values of PD observed.

An interaction between σ and A exists since the curves, for fixed values of the rest of the parameters and different values of A, are not parallel but increasingly diverging as σ and A increase.

As σ increases the curves for a given value of A diverge. This means that, as σ increases, the effect of the absolute level of the rest of the parameters becomes more relevant.

If σ is less than 12 units the values of PD will be less than 20 percent for the ranges of the parameters chosen and for as long as A remains greater than \$50 per order.

Figure 16 shows the graph of the PD versus A for different values of the variable backorder cost, $\hat{\pi}$. The value of PD decreases as A in-



Levels of Parameters

	λ	I	C	π	$\hat{\pi}$	τ
High	3500	0.20	300	0.55	1000	0.10
Med	1850	0.20	155	0.55	675	0.10
Low	400	0.20	8	0.55	350	0.10

Figure 15. Graph of Percent Deviation vs. σ-A (Q,r) Model

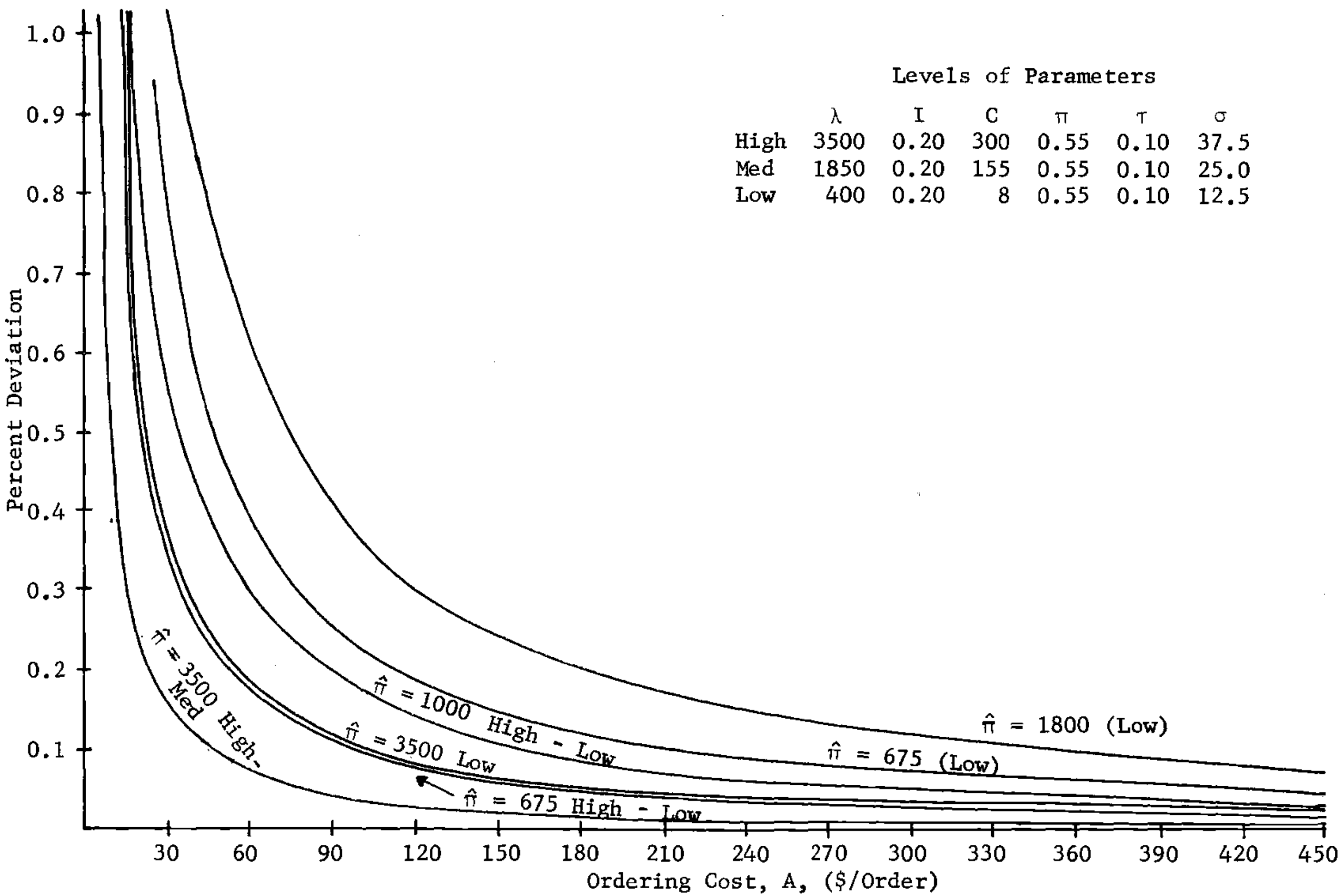


Figure 16. Graph of Percent Deviation vs. A-π̂ (Q,r) Model

creases and $\hat{\pi}$ decreases, and for low values of A very high deviations are always obtained. This relationship can be explained by the same reasons given in explaining the A and $\hat{\pi}$ effects.

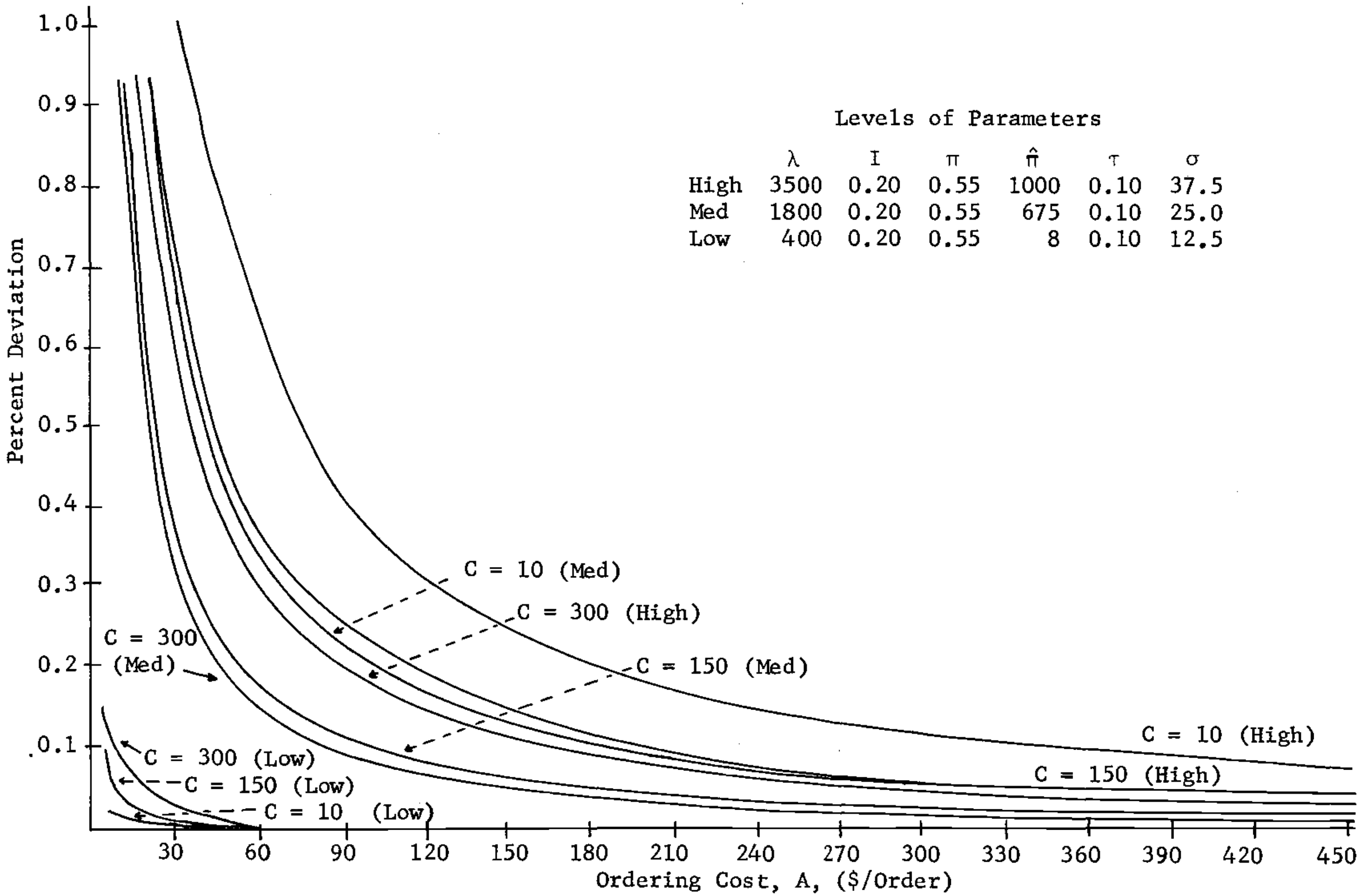
Furthermore, it can be observed that the curves for the high and medium levels for a given value of $\hat{\pi}$ are nearly equal. This means that the effect of the rest of the parameters is negligible for a given value of $\hat{\pi}$ and all values of A as long as they are above the medium level chosen for the study.

Due to the fact that the curves for a fixed level of the rest of the parameters and for different values of $\hat{\pi}$ are not parallel, there exists an interaction between A and $\hat{\pi}$. As A gets very large the interaction becomes less relevant because the curves get more parallel.

The decreasing of the values of PD for a given $\hat{\pi}$ and A as the level of the rest of the parameters decreases can be explained by the fact that for lower levels of λ and C higher deviations are always obtained.

For the ranges used, if A is less than \$15 per order, the PD will be greater than 30 percent regardless of the value of the other parameters. Also, if A is greater than \$345 per order the PD will be less than 10 percent.

The next interaction in order of importance is the AC interaction. The graph of the PD versus this interaction is in Figure 17. As A or C increases the value of PD decreases. Also, as the levels of the other parameters decrease the PD decreases. This pattern of the PD values can be explained using the same arguments as the one used for the A and C effects.



	λ	I	π	$\hat{\pi}$	τ	σ
High	3500	0.20	0.55	1000	0.10	37.5
Med	1800	0.20	0.55	675	0.10	25.0
Low	400	0.20	0.55	8	0.10	12.5

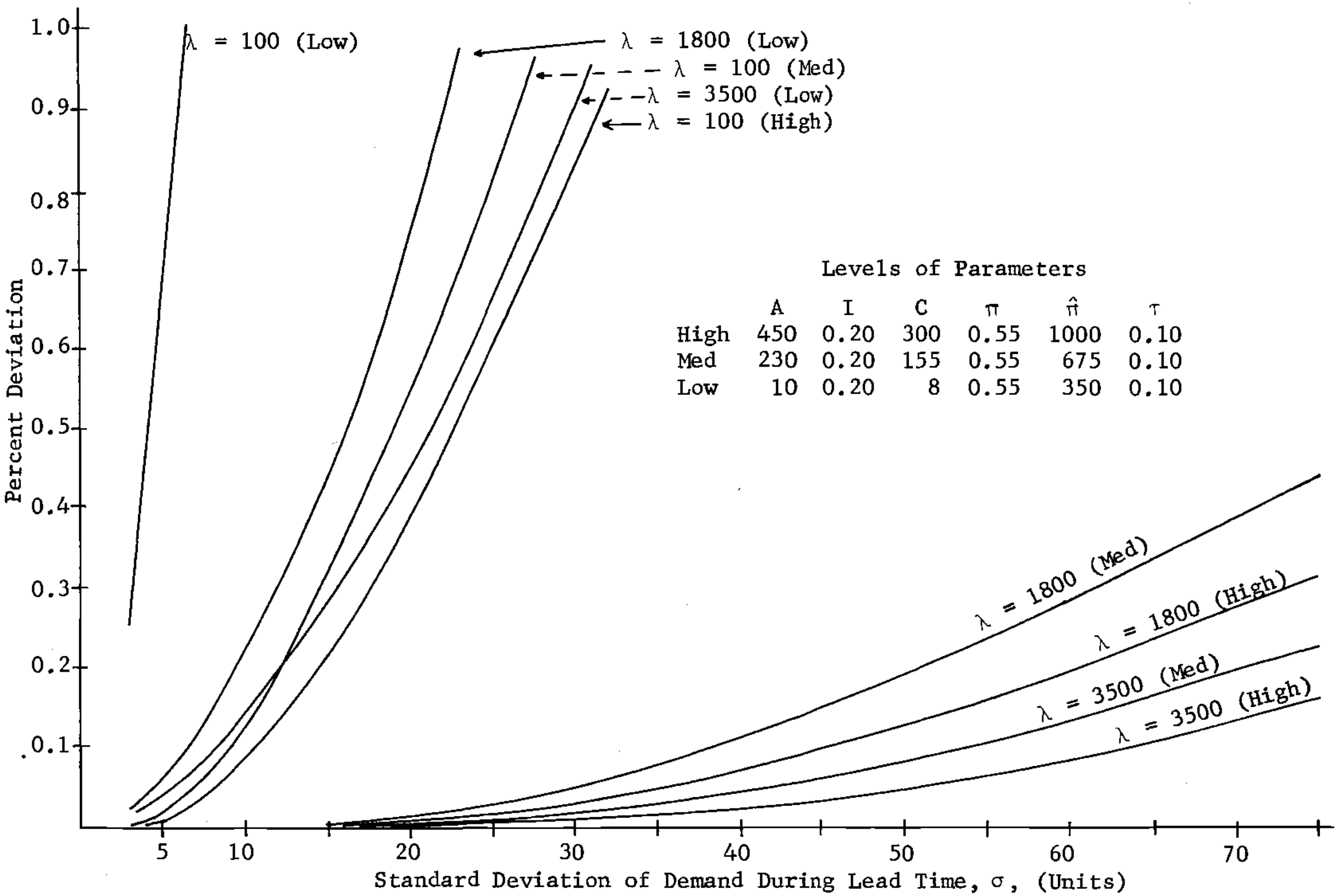
Figure 17. Graph of Percent Deviation vs. AC (Q,r) Model

The very low values of the PD for the low levels of parameters are due to the fact that the value of $\hat{\pi}$ used for this run was \$8 per unit-year of backorders instead of the \$350 per unit-year of backorders used for the low level for all other cases.

The interaction between A and C will become less important as A increases and also as C increases, as can be seen in the graph by the fact that the curves get more parallel as A and C increase.

As the value of A increases the effect of the absolute value of the other parameter becomes less important. This can be seen by the fact that the curves come closer together as A increases. If A is greater than \$180 per order, the value of PD is less than 20 percent for the ranges of parameters chosen.

Next the $\sigma\lambda$ interaction is discussed and Figure 18 shows the graph of PD versus σ for different levels of λ. As σ increases and λ decreases the values of PD increase. This can be explained by the same reasons given for the σ and λ cases. Also, as the level of the other parameters decreases the values of the PD increase. This might be because, as the values of A and C decrease, higher values of the PD are obtained. The huge deviations obtained for the cases where λ is equal to 3500 and 1800 units per year, and the rest of the parameters are at low levels, are due again to the fact that the optimal deterministic policy obtained is the one where no backorders are allowed and A and C are at low levels. For the case where λ is equal to 100 units per year regardless of the level of the other parameters, very high deviations are obtained. This is because here the CV will increase very rapidly as σ increases making the optimal deterministic and stochastic policies differ by amounts that, for this



	A	I	C	π	$\hat{\pi}$	τ
High	450	0.20	300	0.55	1000	0.10
Med	230	0.20	155	0.55	675	0.10
Low	10	0.20	8	0.55	350	0.10

Figure 18. Graph of Percent Deviation vs. $\sigma\lambda$ (Q,r) Model

range of demands, are more significant. As the level of the other parameters decreases, higher deviation will be obtained because again A and C will decrease making the PD grow even more.

The interaction between λ and σ is very prominent. Also, it becomes more relevant as the level of the other parameters decreases.

If σ is less than 37.5 units and the rest of the parameters are not at their low levels, the PD will be less than 10 percent. But, if λ is less than 100 units per year and σ is greater than 15 units, the value of PD will be greater than 20 percent for the ranges chosen for the study.

The graph of the PD versus the $\lambda\hat{\pi}$ interaction is shown in Figure 19. As $\hat{\pi}$ increases and λ decreases, the value of the PD increases. As the rest of the parameters decrease for a given λ and $\hat{\pi}$, the value of the PD increases. For λ at high and medium levels and the rest of the parameters at low levels, the high deviation obtained is because the deterministic policy found does not allow any backorders and C and A are at low level. For λ at the low level, the high deviations obtained are due to the same cause as given in the previous case. The general pattern of this graph can be explained by the same reasons as the λ and $\hat{\pi}$ effects.

Here again the interaction between λ and $\hat{\pi}$ is relevant. It will increase as λ decreases, as is apparent from the fact that the curves get less parallel as λ decreases.

It can be observed, as in the $\hat{\pi}$ effect, that the relation between PD versus $\hat{\pi}$ is linear regardless of the value of the other parameters.

If λ is higher than 1800 units per year and the parameters are at ranges higher than the medium level of this study, the PD will always be less than seven percent for any value of $\hat{\pi}$ in the range chosen.

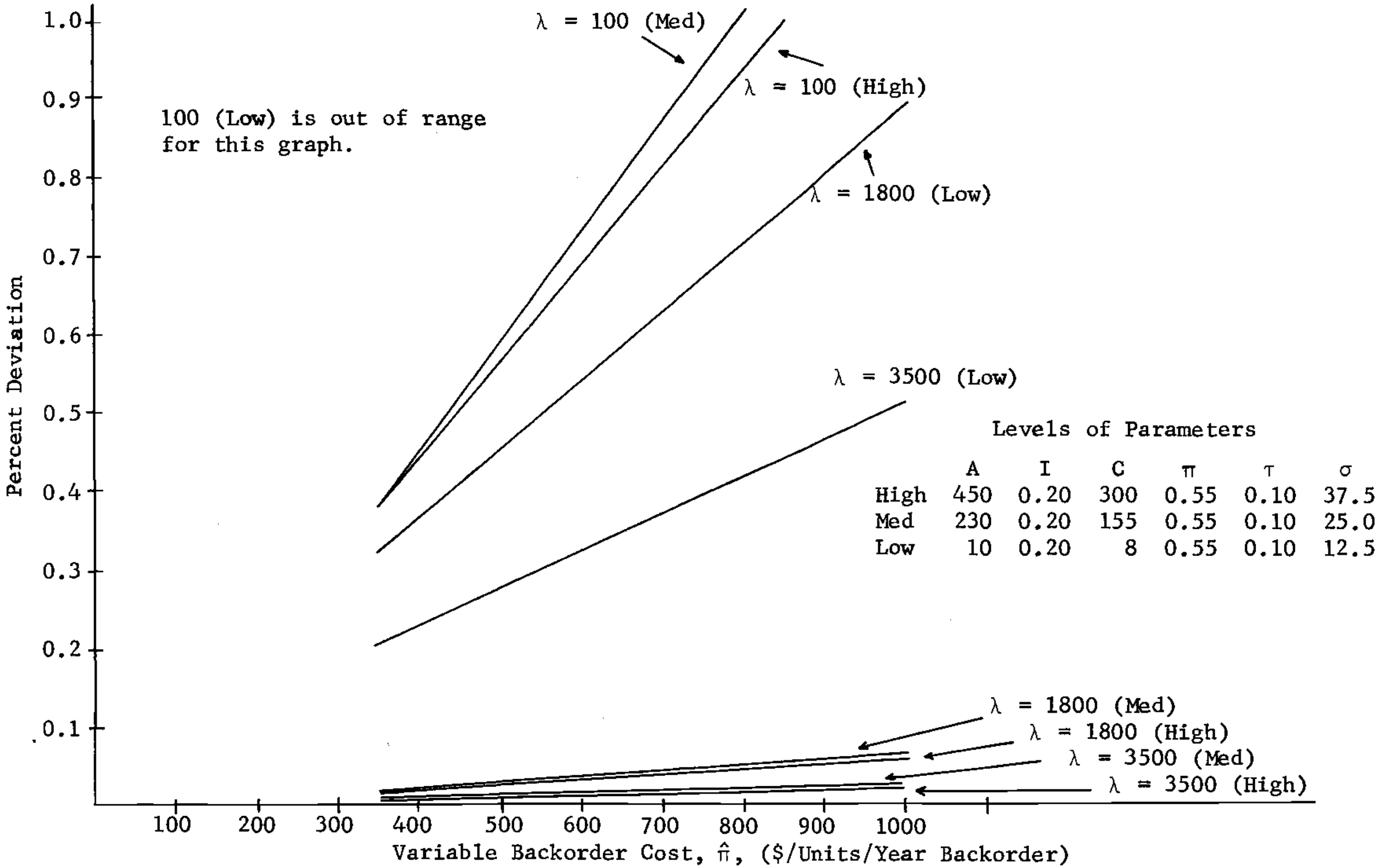
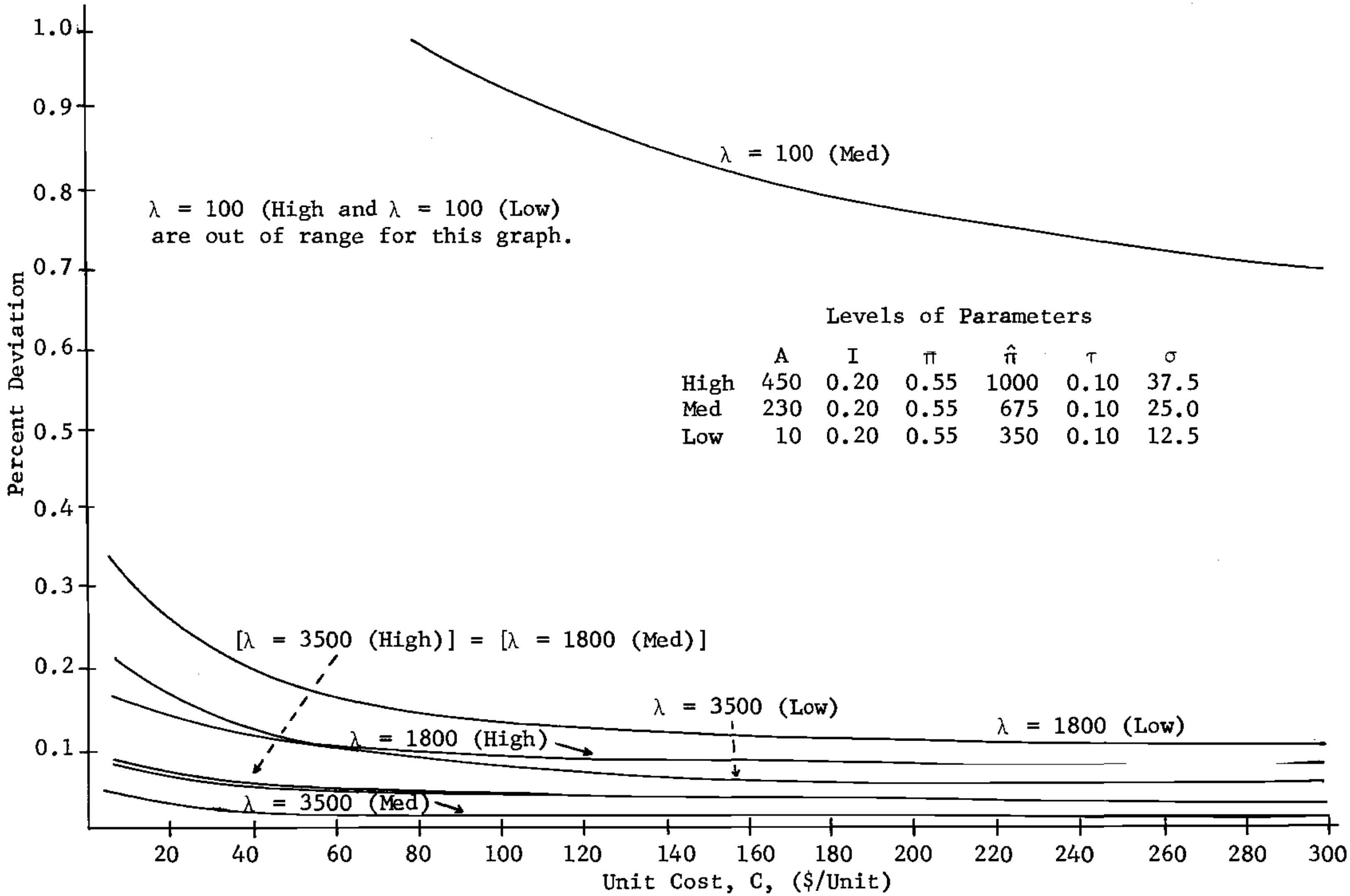


Figure 19. Graph of Percent Deviation vs. $\lambda\hat{\pi}$ (Q,r) Model

The last of the interactions considered relevant was the λC interaction. The graph for this case is shown in Figure 20. As C or λ increases the PD will decrease. This is in accordance with the main effects of λ and C previously studied. The change in the relative position of the low level curve might be due to the same factors as in the C main effect. The high deviations obtained for the cases where λ is at low level are because the predominant effect here is the one due to the level of the CV. For this level it is higher and the huge deviations are obtained.

The interaction between λ and C disappears as C gets large. This can be observed in the tendency of the curves to become parallel as C increases regardless of the value of the rest of the parameters. The effect of C becomes more important as λ decreases.

If C is greater than $40 per unit and the value of λ is greater than 1800 units per year, the PD will be less than 20 percent, but if λ is less than 100 units per year the PD will never be less than 70 percent for the ranges used in the study. A comparison of how each relevant interaction varies for different values of the parameters not belonging to the interaction is given in Table 4. In this table each cell indicates how the importance of the interaction varies as the two parameters involved increase for given levels of the rest of the parameters. In general these interactions did not appear to be very important as can be seen by the fact that all the graphs follow the same pattern as the main effect curves. Therefore, the conditions for the applicability of the deterministic model will be given only in terms of the main effects.



	A	I	π	$\hat{\pi}$	τ	σ
High	450	0.20	0.55	1000	0.10	37.5
Med	230	0.20	0.55	675	0.10	25.0
Low	10	0.20	0.55	350	0.10	12.5

Figure 20. Graph of Percent Deviation vs. λ,C (Q,r) Model

Table 4. Summary of Interaction Effects

Interaction \ Level of Other Param.	LOW	MEDIUM	HIGH	REMARKS
$A\lambda$	Decreases at lower rate	Decreases at higher rate	Decreases at highest rate	As A and λ get large, the interaction tends to decrease
$A\sigma$	Decreases as A increases at low rate	Decreases as A increases at low rate	Decreases as A increases at low rate	All the curves diverge
$A\hat{\pi}$	Approximately constant; for high A almost disappears	No interaction for all values of A	No interaction for all values of A	The high and medium level curves are almost equal; for those levels there is no interaction
AC	Approximately constant; as λ decreases gets less predominant	Approximately constant; as λ decreases gets less relevant	Disappears even at low levels of A	The low PD found for low level curves is because $\hat{\pi}$ is at \$8 per unit-year of backorder
$\lambda\sigma$	As λ increases the interaction decreases	As λ increases the interaction decreases	As λ increases the interaction decreases	All curves diverge
$\lambda\hat{\pi}$	As λ decreases as $\hat{\pi}$ increases highest rate of increase	As λ increases it decreases lower rate	As λ increases it decreases as $\hat{\pi}$ increases it increases lowest rate	All curves are linear
λC	As C gets large the interaction disappears	As C gets large the interaction disappears	As C gets large the interaction disappears	This is the most significant variation in the interaction

In order to obtain more general conclusions, several attempts were made to present the values of the PD as a function of a combination of the most relevant parameters of the system.

The first attempt was to relate the PD to the Wilson lot size, QW. Three cases ouf of the 64 studied were chosen. For each case the values of λ, A, and C were varied so that QW would remain equal to this original value. With these new data sets, a run of the Q program was made to see if the value of the PD remained approximately the same for each of the new cases. This proved to be untrue. A second relationship was tried where the value of the product $\lambda \cdot A \cdot C$ was used and the values of λ, A, and C were varied so that the product would remain constant for each case. Again no general pattern in the values of the PD was found. At this point it was decided that, due to the complexity of the formulations involved, further attempts would be fruitless.

A sample run to show the variation of the PD with respect to the operating cost was made using an arbitrary value of this cost for each of the 64 original set. This run is meant to be an example of this effect and no general conclusions should be drawn from it.

The value of the additional operating cost included in the stochastic model is not supposed to be an absolute quantity but rather a difference in operating costs when the stochastic policy is used to operate the system instead of the deterministic one. The effect of this cost should be analyzed after the value of the PD has been obtained. If the value of this cost is higher than the cost reduction obtained by the use of the stochastic model, then the deterministic model will yield a more economical solution to the system; otherwise, the stochastic model should be used to represent the system. The sample output can be seen in Appendix VI.

Results for R Program

The same solution procedure was not followed with this comparison because a major problem was found with the (R,T) model. The problem was found in the determination of the expected unit year of backorders. When the program was run, the model in some cases gave zero or negative expected unit years of backorders even when the expected number of backorders was sizeable. When the value was negative, the total annual stochastic cost for some cases was less than the corresponding total annual deterministic cost.

The problem was determined to be in equation (22) of the (R,T) model formulation. This equation is the solution to the equation used by Hadley and Whitin (12) to find the expected unit year of backorders when the number of demands per year is normally distributed with mean λt and standard deviation $\sqrt{Dt}$. This equation is

$$U(u,\tau) = \int_0^T \int_u^\infty (\xi - u) \cdot \frac{1}{\sqrt{Dt}} \cdot \phi\left(\frac{\xi - \lambda t}{\sqrt{Dt}}\right) d\xi dt \qquad (30)$$

Equation (30) is a double integral defined only on positive quantities, with limits greater than zero. It is impossible that this equation could yield negative quantities. The analytical development by which equation (22) is derived from equation (30) was checked. It is obtained from a simplification of the exact development of the (nQ,r,t) model, as presented by Hadley and Whitin (12), by setting r equal to R and taking the limit as Q tends to zero of the resulting equations. Then a change of variables is made and an integration by parts is performed. No error could be found in this development, but when certain values for the parameters are substituted into equation (22), negative values of the expected unit years of backorders are obtained for a great number of cases, i.e., for item number 1 where λ was 3500 units per year, A was \$450 per order, I was 0.20, C was \$300 per unit, π was \$1 per unit, $\hat{\pi}$ was \$1000 per unit year of backorder, and τ was 0.1 year, the following results are obtained.

When T is equal to 0.00274 years and R is 351 units, the values of U(R,T) for t equals τ, and t equals τ+T are for D equal to λ.

$$U(R,\tau) = \left[\frac{(3500)^2-2(3500)^4\cdot(0.1)^2}{4\cdot(3500)^3} + \frac{[3500-2\cdot(3500)^2\cdot(0.1)]\cdot 351}{2\cdot(3500)^2} + \frac{(351)^2}{3500}\right]\cdot\Phi\left(\frac{351-3500\cdot(0.1)}{\sqrt{3500\cdot 0.1}}\right) + \frac{1}{2}\left[\sqrt{3500\cdot(0.1)^3} - \frac{\sqrt{(3500)^3\cdot 0.1}}{3500^2} - \frac{\sqrt{3500\cdot 0.1}\cdot 351}{3500}\right]\cdot\phi\left(\frac{351-3500\cdot(0.1)}{\sqrt{3500\cdot 0.1}}\right) - 0$$

$$U(R,\tau) = (17.50-35.05+17.60)\cdot 0.48 + (1.87-0.1-1.88)\cdot 0.40 = 0.2$$

Doing the same for U(R,T+τ) we get U(R,τ+T) = 0.1 ; therefore, the expected number of unit years of backorders B(R,T) is

$$B(R,T) = \frac{1}{T}\,[U(R,\tau+T) - U(R,T)] = \frac{1}{0.00274}\,[0.1 - 0.2] = -\,2.670$$

and the corresponding expected number of backorders for this case was $E(R,T) = 2061.704$.

Also, if $T = 0.00274$ and $R = 361$, the values found will be

$$U(R,\tau+T) = -0.00$$

$$U(R,\tau) = 0.1$$

Therefore

$B(R,T) = -3.934$ for an expected number of backorders of

$E(R,T) = 1336.961$

To avoid this infeasibility, an heuristic rule was included in the model. It consisted of making the expected unit year of backorder zero whenever it was found to be negative by equation (22). This, of course, will give a lower total stochastic cost than the actual one but, as can be seen in the sample output of Appendix V, this was still not a good heuristic rule because for very high values of E(R,T), very low values of B(R,T) are still obtained.

The results from this program will not be valid and will lead to erroneous conclusions, so further investigation of the model was abandoned. As a matter of information, a run was made using the heuristic approximation and the same 64 data used in the Q program. These results are included in Appendix V and are intended to be only of an illustrative nature.

Even though no general conclusions can be drawn about the comparison between the (R,T) model and the deterministic model, it points out an important aspect which was not included in the study. That is, the possibility that even though the stochastic inventory system is represented better by the stochastic model, the resulting total cost and optimal policy might not be the correct or optimal ones due to inadequacies of the stochastic model.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The most important conclusion drawn from this study is that the deterministic model can be used to approximate and economically solve a stochastic inventory system. The cost reduction obtained by the simplicity of the implementation and operation of a deterministic policy will be greater than the additional increase in cost incurred by the less exact formulation of the system.

It can be seen that, if the ordering cost is at high levels, the average demand per year is at high levels, the unit cost is at high levels, the standard deviation of the demand distribution during lead time is small, and the variable backorder cost is small, the approximation will be very good. The situation where all of these requirements are met is not likely to be encountered. Depending on the levels of the factors, it is possible that their effects will be nullified by each other and make the deterministic model the most economical one to use.

The most important factor is the ordering cost. As long as it remains at relatively high values, low values of the PD will occur which indicates that the deterministic model will be a good approximation to use. But, if it is at a low level, the deterministic model is not very likely to be applicable due to the fact that very high deviations will be incurred regardless of the values of the other parameters.

The average demand per year and the standard deviation of the demand during lead time can be analyzed using the coefficient of variation, CV. As long as the coefficient of variation remains at low levels, low values of the PD will be obtained. This of course will favor the applicability of the deterministic model.

The value of the fixed backorder cost, π, has a negligible effect on PD. However, the variable backorder cost, $\hat{\pi}$, is an important factor for the use of the deterministic model. When $\hat{\pi}$ is at very low levels, low deviations will be obtained regardless of the value of the other parameters. But, if it is not at extremely high levels, it will not greatly influence the amount of deviation obtained.

The unit cost, C, will be relevant only if the rest of the parameters are at low levels. When they are at higher levels, the effect of C is negligible.

The value of the procurement lead time, τ, for this study showed no effect on the values of the PD because of the way that σ was defined. It was assumed for the study that the value of σ was independent of τ. In general, this will not be true, as long lead times will often have larger variance associated with them; therefore, the value of τ will have a definitive effect on the values of the PD.

This study can be used also in cases where the lead times are random variables. The effect that this would have on the model would be to increase the amount of variability of the demand during lead time. Because in this study σ was considered an independent parameter, the same model can be used to study this case provided of course that the variation of σ due to this effect could be determined.

This study could also be useful in helping to solve multi-items inventory systems. It may be used as a preliminary study to determine which of the items of the inventory system would need a stochastic policy and which could be operated using a deterministic policy. This would reduce the amount of information and data collection needed to solve the system.

This study pointed out several areas of interest where additional studies could be performed. Further studies can be carried out to determine more general ranges for which deterministic models could be applied to solve stochastic situations. Another possible study could be to try to determine a simple relationship between the relevant parameters so that more general conclusions could be obtained for the values of the PD.

An important fact pointed out by the study was that some models, due to the complexity of the relationship of the variables, might not give valid results as proven by the case of the (R,T) model. This fact favors the applicability of the deterministic models.

In summary, this study has shown that in many cases the inventory policy found using deterministic models is a very good approximation to stochastic inventory models.

Also, when a comparison between stochastic and deterministic models is made, the difference between the operating cost of both policies must be considered. This is an important factor in determining what policy to use since this cost might overrun the cost reduction gained by the use of the stochastic policy.

APPENDICES

APPENDIX I

GIVEN THAT, IF EQUATION (2) HOLDS, ANY VALUE OF S IS OPTIMUM

The total deterministic cost is given by

$$K = \frac{\lambda A}{Q} + \frac{IC}{2Q}(Q-S)^2 + (\pi\lambda S - \frac{\hat{\pi} S^2}{2}) \times \frac{1}{Q} \tag{A-1}$$

for the case when $\hat{\pi} = 0$ and $(\pi\lambda)^2 = 2\lambda AIC$ it is known that the optimal order quantity is

$$Q = \frac{\pi\lambda}{IC} + S$$

Substitution of this quantity into equation (A-1) will give

$$K = \frac{\lambda A}{\frac{\lambda\pi + ICS}{IC}} + \frac{IC}{2(\frac{\pi\lambda + ICS}{IC})}(\frac{\pi\lambda}{IC} + S - S)^2 + (\pi\lambda S + 0) \times \frac{1}{\frac{\pi\lambda + ICS}{IC}}$$

$$K = \frac{\lambda AIC}{\pi\lambda + ICS} + \frac{(IC)^2}{2(\pi\lambda + ICS)} \times \frac{(\pi\lambda)^2}{(IC)^2} + \frac{\pi\lambda ICS}{\pi\lambda + ICS}$$

$$K = \frac{(\pi\lambda)^2}{2(\pi\lambda + ICS)} + \frac{(\pi\lambda)^2}{2(\pi\lambda + ICS)} + \frac{\pi\lambda ICS}{\pi\lambda + ICS}$$

$$K = \frac{(\pi\lambda)^2 + \pi\lambda ICS}{(\pi\lambda + ICS)} = \frac{\pi\lambda(\pi\lambda + ICS)}{(\pi\lambda + ICS)} = \pi\lambda$$

$$K = \pi\lambda$$

Now it has been shown that K is independent of S; therefore, any value of S will be optimal.

APPENDIX II

LISTING OF Q PROGRAM

```
C
C INITIALIZE VARIABLES
C
      DIMENSION A(64),C(64),FPI(64),VPI(64),TAU(64),DI(64),DPY(64),TCW(6
     14),QW(64),U(64),QD(64),SD(64),RD(64),TCD(64)
      DIMENSION QS(64),MU(64),SIG(64),RS(64),TCS(64),TCSQD(64),DEV(64),T
     1CS4(64)
      REAL MU
      LINE=0
C
C READ THE NUMBER OF ITEMS N,FOR WHICH THE INVENTORY POLICIES WILL BE CALCULATED
C
      READ(5,26)N
   26 FORMAT()
C
C READ ADDITIONAL OPERATING COST INCURRED WHEN STOCHASTIC POLICY IS APPLIED TO
C SOLVE THE SYSTEM OCSP
C
      READ(5,113)OCSP
  113 FORMAT(F11.2)
      READ(5,26)M
C
C READ THE FOLLOWING PARAMETERS FOR EACH ITEM:AVERAGE DEMAND PER YEAR DPY,ORDER-
C ING COST A,CARRYING CHARGE DI,UNIT COST C,FIX BACKORDER COST FPI,VARIABLE BACK
C ORDER COST VPI,LEAD TIME TAU,STANDARD DEVIATION OF DEMAND DISTRIBUTION DURING
C LEAD TIME SIG
C
      READ(5,10)(DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),SIG(I),I=1,
     1N)
   10 FORMAT(8F9.2)
C
C DETERMINATION OF THE DETERMINISTIC POLICY FOR EACH ITEM
C
      WRITE(6,01)
   01 FORMAT('1 AVG  DEM   ORDING   CARR    UNIT    FIX BCK  VAR BCK  LE
     1AD   OPT Q   TOT  CST  LEAD TIM   OPT Q    NUM BCK    REORD    TOT
     1CST',/,'  PER YEAR    COST    CHRG    COST    ORD CST  ORD CST  TI
     1ME   WILS    WILSON    DEMAND     DET      ORDER     POINT       DE
     4T',//)
      DO 30 I=1,N
C
C CALCULATION OF Q WILSON QW AND TOTAL COST WILSON TCW
C
      QW(I)=SQRT((2*A(I)*DPY(I))/(DI(I)*C(I)))
      TCW(I)=SQRT(2*DPY(I)*A(I)*DI(I)*C(I))
C
C CALCULATION OF DETERMINISTIC POLICY WHEN BACKORDERS ARE ALLOWED
C
      U(I)=DPY(I)*TAU(I)
      IF(VPI(I).LE.0.0)GO TO 40
      IF(FPI(I).LE.0.0)GO TO 41
      Z=TCW(I)**2*(1+DI(I)*C(I)/VPI(I))-(DI(I)*C(I)/VPI(I))*(FPI(I)*DPY(
     1I))**2
      IF(Z.LT.0.0)GO TO 998
      SD(I)=(-FPI(I)*DPY(I)+SQRT(Z))/(VPI(I)+DI(I)*C(I))
      IF(SD(I).LE.0)GO TO 42
      Y=(2*DPY(I)*A(I))/(DI(I)*C(I))-(FPI(I)*DPY(I))**2/(DI(I)*C(I)*(DI(
     1I)*C(I)+VPI(I)))
      IF(Y.LT.0.0)GO TO 996
      QD(I)=SQRT((VPI(I)+DI(I)*C(I))/VPI(I))*SQRT(Y)
      TCD(I)=(DPY(I)*A(I))/QD(I)+(DI(I)*C(I)*(QD(I)-SD(I))**2)/(2*QD(I))
```

```
      1+(FPI(I)*DPY(I)*SD(I)+(VPI(I)*SD(I)**2)/2)/QD(I)
       RD(I)=U(I)-SD(I)
       GO TO 100
   42 WRITE(6,05)I
   05 FORMAT(' THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QW
     1ILSON FOR ITEM #',I3)
       QD(I)=QW(I)
       SD(I)=0
       TCD(I)=TCW(I)
       RD(I)=U(I)
       LINE=LINE+1
       GO TO 100
C
C SPECIAL CASES OF BACKORDERS MODEL
C
   41 SD(I)=TCW(I)/SQRT(VPI(I)*(DI(I)*C(I)+VPI(I)))
       QD(I)=QW(I)*SQRT((VPI(I)+DI(I)*C(I))/VPI(I))
       RD(I)=U(I)-SD(I)
       TCD(I)=(DPY(I)*A(I))/QD(I)+(DI(I)*C(I)*(QD(I)-SD(I))**2)/(2*QD(I))
      1+(VPI(I)*SD(I)**2)/2*QD(I)
       GO TO 100
   40 IF(FPI(I).LE.0.0)GO TO 43
       IF(ABS(FPI(I)*DPY(I)-TCW(I)).LE.0.000001)GO TO 44
       IF(FPI(I)*DPY(I).LT.TCW(I))GO TO 45
       SD(I)=0
       QD(I)=QW(I)
       TCD(I)=TCW(I)
       RD(I)=U(I)
       GO TO 100
   45 TCD(I)=DPY(I)*FPI(I)
       QD(I)=0
       WRITE(6,12)I
   12 FORMAT(' THE NUMBER OF BACKORDERS IS INFINITE SO AN ORDER IS NEVER
      1 PLACED AND INVENTORY SYSTEM EXISTS FOR ITEM #',I3)
       LINE=LINE+1
       GO TO 100
   44 WRITE(6,13)I
   13 FORMAT(' ANY NUMBER OF BACKORDERS BETWEEN ZERO AND INFINITE SOLVES
      1THE SYSTEM FOR ITEM #',I3)
       LINE=LINE+1
       WRITE (6,14)FPI(I),VPI(I)
   14 FORMAT(' QD=FPI*DPY/DI*C+SD,  RD=U-SD',3X,'FIX PI=',F9.2,3X,'VAR P
      1I=',F9.2)
       LINE=LINE+1
       WRITE(6,15)
   15 FORMAT(' TCD=DPY*A/QD+DI*C*(QD-SD)**2/2*QD+FPI*DPY*SD/QD',//)
       LINE=LINE+1
       GO TO 100
   43 WRITE(6,16)FPI(I),VPI(I),I
   16 FORMAT(' TRIVIAL CASE FOR BACKORDER MODEL',' FIX PI=',F9.2,3X,'VA
      1R PI=',F9.2,' ITEM #',I3)
       LINE=LINE+1
C
C WRITE OUT THE OPTIMAL DETERMINISTIC POLICY
C
  100 WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
      1),U(I),QD(I),SD(I),RD(I),TCD(I),I
```

```
   17 FORMAT(F9.1,2X,F8.2,2X,F4.2,1X,F8.2,2X,F7.2,2X,F8.2,2X,F5.2,1X,F8.
     12,1X,F9.2,2X,F7.1,1X,F9.2,1X,F8.2,1X,F8.2,1X,F10.2,2X,I3)
      LINE=LINE+1
      IF(LINE.GT.45) GO TO 1234
      GO TO 30
  998 WRITE(6,997)I
  997 FORMAT(' THE NUMBER OF BACKORDERS IS AN IMAGINARY NUMBER SO THERE
     1IS NO REAL SOLUTION FOR ITEM #',I3)
      WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
     1),U(I),QD(I),SD(I),RD(I),TCD(I),I
      LINE=LINE+2
      IF(LINE.GT.45) GO TO 1234
      GO TO 30
  996 WRITE(6,995)I
  995 FORMAT(' QD IS AN IMAGINARY NUMBER SO THERE IS NO REAL SOLUTIOM FO
     1R ITEM #',I3)
      WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
     1),U(I),QD(I),SD(I),RD(I),TCD(I),I
      LINE=LINE+2
      IF(LINE.GT.45) GO TO 1234
      GO TO 30
 1234 WRITE(6,01)
      LINE=0
   30 CONTINUE
C
C DETERMINATION OF THE STOCHACHASTIC POLICY FOR EACH ITEM
C
      LINE=0
      WRITE(6,02)
   02 FORMAT(1H1,' STDV OF DEM  MEAN OF DEM   OPT Q        REORD        TO
     1T CST',/,'  DTR LEAD TM  DTR LEAD TM    STOC         POINT            S
     1TOC   ITEM #',//)
      DO 31 I=1,N
C
C INITIALIZE VARIABLES
C
      IF(QD(I).LE.0.0)GO TO 989
      QS(I)=QW(I)
      RS(I)=RD(I)
      MU(I)=U(I)
C
C LOOP TO FIND OPTIMAL R FOR A GIVEN VALUE OF Q
C
C GROSS SEARCH FOR THE OPTIMAL R FOR A GIVEN Q
C
C FIND TERM INDEPENDENT OF R CALLED F
C
  999 F=QS(I)*DI(I)*C(I)
      DELTA=1.00
C
C FIND CUMULATIVE DENSITY FUNCTION CDF AND PROBABILITY DENSITY FUNCTION PDF FOR
C A STANDARD NORMAL RANDOM VARIABLE X
C
  101 X=(RS(I)-MU(I))/SIG(I)
      GO TO 957
  951 CDF=0.0
      GO TO 952
```

```
      953 CDF=1.0
          GO TO 952
      955 PDF=0.0
          GO TO 956
C
C IF X IS LESS THAN -5 THEN CDF=1 AND IF X IS GREATER THAN 5 THEN CDF=0
C
      957 IF(X.GE.5.0)GO TO 951
          IF(X.LE.-5.0)GO TO 953
          CDF1=RNORM(X)
          CDF=1-CDF1
C
C IF ABSOLUTE VALUE OF X IS GREATER THAN 13 THEN PDF=0
C
      952 IF(ABS(X).GE.13.0)GO TO 955
          PDF=1/(EXP(X**2/2)*2.5066)
C
C FIND TERM DEPENDING ON R CALLED B
C
      956 B=(VPI(I)+DI(I)*C(I))*(SIG(I)*PDF-(RS(I)-MU(I))*CDF)+FPI(I)*OPY(I)
         1*CDF
C
C COMPARE TO SEE IF B IS LESS THAN F
C
          IF(B.LT.F)GO TO 46
          RS(I)=RS(I)+DELTA
          GO TO 101
C
C FINE SEARCH FOR THE OPTIMAL R FOR A GIVEN Q
C
       46 RS(I)=RS(I)-DELTA
          DELTA=DELTA/100.00
      102 RS(I)=RS(I)+DELTA
C
C FIND CUMULATIVE DENSITY FUNCTION CDF AND PROBABILITY DENSITY FUNCTION PDF FOR
C A STANDARD NORMAL RANDOM VARIABLE X
C
          X=(RS(I)-MU(I))/SIG(I)
          GO TO 977
      971 CDF=0.0
          GO TO 972
      973 CDF=1.0
          GO TO 972
      975 PDF=0.0
          GO TO 976
C
C IF X IS LESS THAN -5 THEN CDF=1 AND IF X IS GREATER THAN 5 THEN CDF=0
C
      977 IF(X.GE.5.0)GO TO 971
          IF(X.LE.-5.0)GO TO 973
          CDF1=RNORM(X)
          CDF=1-CDF1
C
C IF ABSOLUTE VALUE OF X IS GREATER THAN 13 THEN PDF=0
C
      972 IF(ABS(X).GE.13.0)GO TO 975
          PDF=1/(EXP(X**2/2)*2.5066)
```

```
  976 B=(VPI(I)+DI(I)*C(I))*(SIG(I)*PDF-(RS(I)-MU(I))*COF)+FPI(I)*DPY(I)
     1*COF
C
C COMPARE TO DETERMINE IF B AND F ARE EQUAL IF SO GET OUT OF THE LOOP
C
      IF(F-B.GT.0)GO TO 47
      GO TO 102
C
C WITH THIS OPTIMUN R FIND ANOTHER Q
C
   47 ALP=SIG(I)*PDF-(RS(I)-MU(I))*COF
      BET=(SIG(I)**2+(RS(I)-MU(I))**2)*COF/2-SIG(I)*(RS(I)-MU(I))*PDF/2
      QS12=SQRT((2*DPY(I)*(A(I)+FPI(I)*ALP)+2*(VPI(I)+DI(I)*C(I))*BET)/(
     1DI(I)*C(I)))
C
C COMPARE WITH PREVIOUS Q IF EQUAL STOP IF NOT GO BACK AND FIND OPTIMAL R FOR
C NEW Q
C
      IF(ABS(QS12-QS(I)).LE.0.01)GO TO 48
      QS(I)=QS12
      GO TO 999
C
C FIND TOTAL COST FOR Q AND R OPTIMALS CALLED TCS
C
   48 QS(I)=QS12
      IF(MU(I)+2.0*SIG(I).GT.QS(I)+RS(I))GO TO 994
      TCS(I)=DPY(I)*A(I)/QS(I)+DI(I)*C(I)*(QS(I)/2+RS(I)-MU(I))+(FPI(I)*
     1DPY(I)*ALP)/QS(I)+(VPI(I)+DI(I)*C(I))*BET/QS(I)
C
C WRITE OPTIMUN STOCHASTIC POLICY FOR EACH ITEM
C
      WRITE(6,19)SIG(I),MU(I),QS(I),RS(I),TCS(I),I
   19 FORMAT(1X,F8.1,4X,F8.1,4X,F9.2,4X,F8.2,3X,F10.2,3X,I3)
      LINE=LINE+1
      IF(LINE.GT.45) GO TO 1235
      GO TO 31
  994 WRITE(6,993)I
  993 FORMAT(' FOR ITEM #',I3,' THERE IS A POSITIVE PROBABILITY THAT THE
     1 LEAD TIME DEMAND BE GREATER THAN Q+R',/,' SO FOR THIS CASE THE MO
     2DEL IS NOT APPLICABLE')
      WRITE(6,19)SIG(I),MU(I),QS(I),RS(I),TCS(I),I
      LINE=LINE+3
      IF(LINE.GT.45) GO TO 1235
      GO TO 31
  989 WRITE(6,987)I
  987 FORMAT(' THIS ITEM HAS NO SOLUTION IN THE DETERMINISTIC PART',10X,
     1I3)
      LINE=LINE+1
      IF(LINE.GT.45) GO TO 1235
      GO TO 31
 1235 WRITE(6,02)
      LINE=0
   31 CONTINUE
C
C CALCULATION OF PERCENT DEVIATION FOR ALL ITEMS IF WE USE THE OPTIMAL DETERMI-
C NISTIC POLICY INSTEAD OF THE OPTIMAL STOCHASTIC ONE
C
```

```
      LINE=0
      WRITE(6,03)
   03 FORMAT(1H1,'   OPT Q     REORD PT      OPT Q    REORD PT      TOT CST
     1 TOT CST    TOT CST STC    PERCENT',/,'    DETRM        DETRM         STOCH
     2     STOCH          DETRM          STOCH       Q,R DETRM    DEVIATION  ITEM
     3',//)
      DO 33 I=1,N
      IF(QD(I).LE.0.0)GO TO 889
      IF(MU(I)+2.0*SIG(I).GT.QS(I)+RS(I))GO TO 992
C
C CALCULATION OF CDF AND PDF AS BEFORE BUT USING THE OPTIMAL DETERMINISTIC
C POLICY
C
      X=(RD(I)-MU(I))/SIG(I)
      GO TO 857
  851 CDF=0.0
      GO TO 852
  853 CDF=1.0
      GO TO 852
  855 PDF=0.0
      GO TO 856
C
C IF X IS LESS THAN -5 THEN CDF=1 AND IF X IS GREATER THAN 5 THEN CDF=0
C
  857 IF(X.GE.5.0)GO TO 851
      IF(X.LE.-5.0)GO TO 853
      CDF1=RNORM(X)
      CDF=1-CDF1
C
C IF ABSOLUTE VALUE OF X IS GREATER THAN 13 THEN PDF=0
C
  852 IF(ABS(X).GE.13.0)GO TO 855
      PDF=1/(EXP(X**2/2)*2.5066)
C
C FIND TOTAL COST USING THE OPTIMAL DETERMINISTIC POLICY TCSQD
C
  856 ALP=SIG(I)*PDF-(RD(I)-MU(I))*CDF
      BET=(SIG(I)**2+(RD(I)-MU(I))**2)*CDF/2-SIG(I)*(RD(I)-MU(I))*PDF/2
      TCSQD(I)=DPY(I)*A(I)/QD(I)+DI(I)*C(I)*(QD(I)/2+RD(I)-MU(I))+(FPI(I
     1)*DPY(I)*ALP)/QD(I)+(VPI(I)+C(I)*DI(I))*BET/QD(I)
C
C CALCULATION OF PERCENT DEVIATION DEV
C
      DEV(I)=(TCSQD(I)-TCS(I))/TCS(I)
C
C WRITE OUT VALUES OF PERCENT DEVIATION FOR EACH ITEM
C
      WRITE(6,04)QD(I),RD(I),QS(I),RS(I),TCD(I),TCS(I),TCSQD(I),DEV(I),I
   04 FORMAT(4(F9.2,1X),3(F11.2,1X),F11.4,3X,I3)
      LINE=LINE+1
      IF(LINE.GT.45) GO TO 1236
      GO TO 33
  992 WRITE(6,993)I
      WRITE(6,04)QD(I),RD(I),QS(I),RS(I),TCD(I),TCS(I),TCSQD(I),DEV(I),I
      LINE=LINE+3
      IF(LINE.GT.45) GO TO 1236
      GO TO 33
```

```
      889 WRITE(6,985)I
      985 FORMAT(' THIS ITEM HAS NO SOLUTION IN THE DETERMINISTIC PART',58X,
         1I3)
          LINE=LINE+1
          IF(LINE.GT.45) GO TO 1236
          GO TO 33
     1236 WRITE(6,03)
          LINE=0
       33 CONTINUE
C
C CALCULATION OF PERCENT DEVIATION ADDING TO THE TOTAL STOCHASTIC COST THE
C ADDITIONAL COST FOR OPERATING THE SYSTEM WHEN THE STOCHASTIC POLICY IS USED TO
C OPERATE THE SYSTEM
C
          LINE=0
          COSP=OCSP
          DO 5000 K=1,M
          WRITE(6,5004)
     5004 FORMAT(1H1,'  OPT Q     REORD PT     OPT Q    REORD PT     TOT CST  TO
         1T CST STC   TOT CST STC  OPERATING     PERCENT',/,'    DETRM      DETR
         2M      STOCH      STOCH          DETRM    WITH OPR CST  Q,R DETRM     C
         3OST      DEVIATION  ITEM #',//)
          OCSP=COSP**K
          DO 5001 I=1,N
          IF(QD(I).LE.0.0)GO TO 5002
          IF(MU(I)+2.0*SIG(I).GT.QS(I)+RS(I))GO TO 5005
          TCS4(I)=TCS(I)+OCSP
          DEV(I)=(TCSQD(I)-TCS4(I))/TCS4(I)
          WRITE(6,94)QD(I),RD(I),QS(I),RS(I),TCD(I),TCS4(I),TCSQD(I),OCSP,DE
         1V(I),I
       94 FORMAT(4(F9.2,1X),4(F11.2,1X),F11.4,3X,I3)
          LINE=LINE+1
          IF (LINE.GT.45)GO TO 1237
          GO TO 5001
     5005 WRITE(6,993)I
          WRITE(6,94)QD(I),RD(I),QS(I),RS(I),TCD(I),TCS4(I),TCSQD(I),OCSP,DE
         1V(I),I
          LINE=LINE+3
          IF (LINE.GT.45)GO TO 1237
          GO TO 5001
     5002 WRITE(6,5006)I
     5006 FORMAT(' THIS ITEM HAS NO SOLUTION IN THE DETERMINISTIC PART',50X,
         1I3)
          LINE=LINE+1
          IF (LINE.GT.45)GO TO 1237
          GO TO 5001
     1237 WRITE(6,5004)
          LINE=0
     5001 CONTINUE
          LINE=0
     5000 CONTINUE
          CALL EXIT
          END

END OF COMPILATION:       NO  DIAGNOSTICS.
```

APPENDIX III

LISTING OF R PROGRAM

```
C
C INITIALIZE VARIABLES
C
      DIMENSION A(64),C(64),FPI(64),VPI(64),TAU(64),DI(64),DPY(64),TCW(6
     14),QW(64),U(64),QD(64),SD(64),RD(64),TCD(64)
      DIMENSION R(64),T(64),TCSRD(64),TCS1(64),DEV(64),MU(64),ENBO(64),E
     1UYBO(64),UTAUT(64),UTAU(64),P(400),TCS2(400),IT(64),TCS3(600),TS(6
     24),RS(64),TCS4(64)
      REAL MU,MEA1,MEA2
      LINE=0
C
C READ THE NUMBER OF ITEMS N,FOR WHICH THE INVENTORY POLICIES WILL BE CALCULATED
C
      READ(5,26)N
   26 FORMAT()
C
C READ ADDITIONAL OPERATING COST INCURRED WHEN STOCHASTIC POLICY IS APPLIED TO
C SOLVE THE SYSTEM OCSP
C
      READ(5,113)OCSP
  113 FORMAT(F11.2)
      READ(5,26)M
C
C READ THE FOLLOWING PARAMETERS FOR EACH ITEM:AVERAGE DEMAND PER YEAR DPY,ORDER-
C ING COST A,CARRYING CHARGE DI,UNIT CUST C,FIX BACKORDER COST FPI,VARIABLE BACK
C ORDER COST VPI,LEAD TIME TAU
C
      READ(5,10)(DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),I=1,N)
   10 FORMAT(7F9.2)
C
C DETERMINATION OF THE DETERMINISTIC POLICY FOR EACH ITEM
C
      WRITE(6,01)
   01 FORMAT('1 AVG  DEM   ORDING   CARR    UNIT    FIX BCK  VAR BCK  LE
     1AD   OPT Q   TOT  CST  LEAD TIM   OPT Q   NUM BCK   REORD    TOT
     1CST',/,'  PER YEAR    COST     CHRG     COST     ORD CST  ORD CST  TI
     1ME    WILS    WILSON    DEMAND     DET      ORDER     POINT       DE
     4T',//)
      DO 30 I=1,N
C
C CALCULATION OF Q WILSON QW AND TOTAL COST WILSON TCW
C
      QW(I)=SQRT((2*A(I)*DPY(I))/(DI(I)*C(I)))
      TCW(I)=SQRT(2*DPY(I)*A(I)*DI(I)*C(I))
C
C CALCULATION OF DETERMINISTIC POLICY WHEN BACKORDERS ARE ALLOWED
C
      U(I)=DPY(I)*TAU(I)
      IF(VPI(I).LE.0.0)GO TO 40
      IF(FPI(I).LE.0.0)GO TO 41
```

```
      Z=TCW(I)**2*(1+DI(I)*C(I)/VPI(I))-(DI(I)*C(I)/VPI(I))*(FPI(I)*DPY(
     1I))**2
      IF(Z.LT.0.0)GO TO 998
      SD(I)=(-FPI(I)*DPY(I)+SQRT(Z))/(VPI(I)+DI(I)*C(I))
      IF(SD(I).LE.0)GO TO 42
      Y=(2*DPY(I)*A(I))/(DI(I)*C(I))-(FPI(I)*DPY(I))**2/(DI(I)*C(I)*(DI(
     1I)*C(I)+VPI(I)))
      IF(Y.LT.0.0)GO TO 996
      QD(I)=SQRT((VPI(I)+DI(I)*C(I))/VPI(I))*SQRT(Y)
      TCD(I)=(DPY(I)*A(I))/QD(I)+(DI(I)*C(I)*(QD(I)-SD(I))**2)/(2*QD(I))
     1+(FPI(I)*DPY(I)*SD(I)+(VPI(I)*SD(I)**2)/2)/QD(I)
      RD(I)=U(I)-SD(I)
      GO TO 100
   42 WRITE(6,05)I
   05 FORMAT(' THE NUMBER OF BACKORDERS SD IS NEGATIVE SO SD=0 AND QD=QW
     1ILSON FOR ITEM #',I3)
      QD(I)=QW(I)
      SD(I)=0
      TCD(I)=TCW(I)
      RD(I)=U(I)
      LINE=LINE+1
      GO TO 100
C
C SPECIAL CASES OF BACKORDERS MODEL
C
   41 SD(I)=TCW(I)/SQRT(VPI(I)*(DI(I)*C(I)+VPI(I)))
      QD(I)=QW(I)*SQRT((VPI(I)+DI(I)*C(I))/VPI(I))
      RD(I)=U(I)-SD(I)
      TCD(I)=(DPY(I)*A(I))/QD(I)+(DI(I)*C(I)*(QD(I)-SD(I))**2)/(2*QD(I))
     1+(VPI(I)*SD(I)**2)/2*QD(I)
      GO TO 100
   40 IF(FPI(I).LE.0.0)GO TO 43
      IF(ABS(FPI(I)*DPY(I)-TCW(I)).LE.0.000001)GO TO 44
      IF(FPI(I)*DPY(I).LT.TCW(I))GO TO 45
      SD(I)=0
      QD(I)=QW(I)
      TCD(I)=TCW(I)
      RD(I)=U(I)
      GO TO 100
   45 TCD(I)=DPY(I)*FPI(I)
      QD(I)=0
      WRITE(6,12)I
   12 FORMAT(' THE NUMBER OF BACKORDERS IS INFINITE SO AN ORDER IS NEVER
     1 PLACED AND INVENTORY SYSTEM EXISTS FOR ITEM #',I3)
      LINE=LINE+1
      GO TO 100
   44 WRITE(6,13)I
   13 FORMAT(' ANY NUMBER OF BACKORDERS BETWEEN ZERO AND INFINITE SOLVES
     1THE SYSTEM FOR ITEM #',I3)
      LINE=LINE+1
      WRITE (6,14)FPI(I),VPI(I)
   14 FORMAT(' QD=FPI*DPY/DI*C+SD,  RD=U-SD',3X,'FIX PI=',F9.2,3X,'VAR P
     1I=',F9.2)
      LINE=LINE+1
      WRITE(6,15)
   15 FORMAT(' TCD=DPY*A/QD+DI*C*(QD-SD)**2/2*QD+FPI*DPY*SD/QD',//)
      LINE=LINE+1
```

```
          GO TO 100
       43 WRITE(6,16)FPI(I),VPI(I),I
       16 FORMAT(' TRIVIAL CASE FOR BACKORDER MODEL',' FIX PI=',F9.2,3X,'VA
         1R PI=',F9.2,' ITEM #',I3)
          LINE=LINE+1
C
C WRITE OUT THE OPTIMAL DETERMINISTIC POLICY
C
  100 WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
     1),U(I),QD(I),SD(I),RD(I),TCD(I),I
   17 FORMAT(F9.1,2X,F8.2,2X,F4.2,1X,F8.2,2X,F7.2,2X,F8.2,2X,F5.2,1X,F8.
     12,1X,F9.2,2X,F7.1,1X,F9.2,1X,F8.2,1X,F8.2,1X,F10.2,2X,I3)
          LINE=LINE+1
          IF(LINE.GT.45) GO TO 1234
          GO TO 30
  998 WRITE(6,997)I
  997 FORMAT(' THE NUMBER OF BACKORDERS IS AN IMAGINARY NUMBER SO THERE
     1IS NO REAL SOLUTION FOR ITEM #',I3)
      WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
     1),U(I),QD(I),SD(I),RD(I),TCD(I),I
          LINE=LINE+2
          IF(LINE.GT.45) GO TO 1234
          GO TO 30
  996 WRITE(6,995)I
  995 FORMAT(' QD IS AN IMAGINARY NUMBER SO THERE IS NO REAL SOLUTIOM FO
     1R ITEM #',I3)
      WRITE(6,17)DPY(I),A(I),DI(I),C(I),FPI(I),VPI(I),TAU(I),QW(I),TCW(I
     1),U(I),QD(I),SD(I),RD(I),TCD(I),I
          LINE=LINE+2
          IF(LINE.GT.45) GO TO 1234
          GO TO 30
 1234 WRITE(6,01)
          LINE=0
   30 CONTINUE
C
C DETERMINATION OF THE STOCHASTIC POLICY FOR EACH ITEM
C
          LINE=0
          WRITE(6,704)
  704 FORMAT(1H1,'  STDV DEM  MEAN DEM  STDV DEM  MEAN DEM    EXP NUM  E
     1XP UNT-   OPT CYLE   ORD UP TO   TOT CST',/,'   DIST TAU  DIST TAU
     2  DIS TAU+T DIS TAU+T   BACKORD   YR BACKOR  TIME STCH  LEVEL STC
     3  STOCH      ITEM #',///)
          DO 301 I=1,N
C
C INITIALIZE VARIABLES
C
          IF (QD(I).LT.0.0)GO TO 1989
          DELTT=0.0274
          J=1
          MU(I)=U(I)
          T(I)=0.00274
C
C SEARCH TO FIND OPTIMAL ORDER UP TO LEVEL R FOR A GIVEN INTER REVIEW TIME T
C
C
C GROSS SEARCH ON R FOR A GIVEN T
```

```
C
  302 R(I)=MU(I)
 1302 K=1
      KK=1
      DELTA=10.
C
C CALCULATE MEAN AND STANDARD DEVIATION OF DEMAND DISTRIBUTION DURING LEAD TIME
C AND MEAN AND STANDARD DEVIATION OF DEMAND DISTRIBUTION DURING LEAD TIME PLUS
C INTERREVIEW TIME
C
      SIG1=SQRT(DPY(I)*TAU(I))
      SIG2=SQRT(DPY(I)*(TAU(I)+T(I)))
      MEA1=DPY(I)*TAU(I)
      MEA2=DPY(I)*(TAU(I)+T(I))
C
C DETERMINE CUMULATIVE DENSITY FUNCTION AND PROBABILITY DENSITY FUNCTION FOR
C FOUR STANDARD NORMAL RANDOM VARIABLES X1,X2,X3,X4
C
  304 X1=(R(I)-MEA1)/SIG1
      X2=(R(I)-MEA2)/SIG2
      X3=(R(I)+MEA1)/SIG1
      X4=(R(I)+MEA2)/SIG2
C
C IF X GREATER THAN 5 CDF=0 AND IF X LESS THAN -5 CDF=1
C
      IF(X1.GE.5.)GO TO 313
      IF(X1.LE.-5.)GO TO 314
      TCDF1=RNORM(X1)
      CDF1=1-TCDF1
  321 IF(X2.GE.5.)GO TO 315
      IF(X2.LE.-5.)GO TO 316
      TCDF2=RNORM(X2)
      CDF2=1-TCDF2
  322 IF(X3.GE.5.)GO TO 317
      TCDF3=RNORM(X3)
      CDF3=1-TCDF3
  323 IF(X4.GE.5.)GO TO 318
      TCDF4=RNORM(X4)
      CDF4=1-TCDF4
C
C IF ABSOLUTE VALUE OF X GREATER THAN 13 PDF=0
C
  324 IF(ABS(X1).GE.13.)GO TO 319
      PDF1=1/(EXP(X1**2/2)*2.5066)
  325 IF(ABS(X2).GE.13.)GO TO 320
      PDF2=1/(EXP(X2**2/2)*2.5066)
  326 IF(ABS(CDF3-CDF4).LE.0.000001)GO TO 351
      GO TO 1326
  313 CDF1=0
      GO TO 321
  314 CDF1=1
      GO TO 321
  315 CDF2=0
      GO TO 322
  316 CDF2=1
      GO TO 322
  317 CDF3=0
```

```
      GO TO 323
  318 CDF4=0
      GO TO 324
  319 PDF1=0
      GO TO 325
  320 PDF2=0
      GO TO 326
C
C CALCULATION OF THE EXPECTED UNIT YEAR OF BACKORDER
C
 1326 Z1=(DPY(I)**2+2*DPY(I)**4*TAU(I)**2)/(4*DPY(I)**3)
      Z2=(DPY(I)-2.0*DPY(I)**2*TAU(I))*R(I)/(2.0*DPY(I)**2)
      Z3=R(I)**2/(2*DPY(I))
      Z4=SQRT(DPY(I))*SQRT(TAU(I)**3)
      Z5=SQRT(DPY(I)**3)*SQRT(TAU(I))/DPY(I)**2
      Z6=SIG1*R(I)/DPY(I)
      UTAU(I)=(Z1+Z2+Z3)*CDF1+0.5*(Z4-Z5-Z6)*PDF1-1/(4*DPY(I))*EXP(2*R(I
     1))*CDF3
      IF(UTAU(I).LT.0.0)UTAU(I)=0.0
      Z7=(DPY(I)**2+2*DPY(I)**4*(TAU(I)+T(I))**2)/(4*DPY(I)**3)
      Z8=(DPY(I)-2*DPY(I)**2*(TAU(I)+T(I))*R(I))/(2*DPY(I)**2)
      Z9=R(I)**2/(2*DPY(I))
      Z10=SQRT(DPY(I))*SQRT((TAU(I)+T(I))**3)
      Z11=SQRT(DPY(I)**3)*SQRT(TAU(I)+T(I))/DPY(I)**2
      Z12=SIG2*R(I)/DPY(I)
      UTAUT(I)=(Z7+Z8+Z9)*CDF2+0.5*(Z10-Z11-Z12)*PDF2-EXP(2*R(I))*CDF4/(
     14*DPY(I))
      IF(UTAUT(I).LT.0.0)UTAUT(I)=0.0
      GO TO 352
  351 Z1=(DPY(I)**2+2*DPY(I)**4*TAU(I)**2)/(4*DPY(I)**3)
      Z2=(DPY(I)-2.0*DPY(I)**2*TAU(I))*R(I)/(2.0*DPY(I)**2)
      Z3=R(I)**2/(2*DPY(I))
      Z4=SQRT(DPY(I))*SQRT(TAU(I)**3)
      Z5=SQRT(DPY(I)**3)*SQRT(TAU(I))/DPY(I)**2
      Z6=SIG1*R(I)/DPY(I)
      UTAU(I)=(Z1+Z2+Z3)*CDF1+0.5*(Z4-Z5-Z6)*PDF1
      IF(UTAU(I).LT.0.0)UTAU(I)=0.0
      Z7=(DPY(I)**2+2*DPY(I)**4*(TAU(I)+T(I))**2)/(4*DPY(I)**3)
      Z8=(DPY(I)-2*DPY(I)**2*(TAU(I)+T(I))*R(I))/(2*DPY(I)**2)
      Z9=R(I)**2/(2*DPY(I))
      Z10=SQRT(DPY(I))*SQRT((TAU(I)+T(I))**3)
      Z11=SQRT(DPY(I)**3)*SQRT(TAU(I)+T(I))/DPY(I)**2
      Z12=SIG2*R(I)/DPY(I)
      UTAUT(I)=(Z7+Z8+Z9)*CDF2+0.5*(Z10-Z11-Z12)*PDF2
      IF(UTAUT(I).LT.0.0)UTAUT(I)=0.0
  352 EUYBO(I)=1/T(I)*(UTAUT(I)-UTAU(I))
      IF(EUYBO(I).LT.0.0)EUYBO(I)=0.0
C
C CALCULATION OF THE EXPECTED NUMBER OF BACKORDERS
C
      ENBO(I)=1/T(I)*(SIG2*PDF2-(R(I)-MEA2)*CDF2-SIG1*PDF1+(R(I)-MEA1)*C
     1DF1)
C
C CALCULATION OF TOTAL COST
C
      TCS3(K)=A(I)/T(I)+DI(I)*C(I)*(R(I)-MU(I)-DPY(I)*T(I)/2)+FPI(I)*ENB
     1O(I)+(DI(I)*C(I)+VPI(I))*EUYBO(I)
```

```
      C
      C COMPARISON OF TOTAL COST TO DETERMINE CHANGE OF DIRECTION AND OPTIMAL R FOR A
      C GIVEN T
      C
            IF(K.EQ.1)GO TO 400
            IF(TCS3(K).GT.TCS3(K-1).AND.DELTA-0.1.LT.0.001)GO TO 491
        592 IF(TCS3(K).GT.TCS3(K-1))GO TO 303
        400 R(I)=R(I)+DELTA
            K=K+1
            KK=KK+1
            GO TO 304
      C
      C FINE SEARCH ON R FOR A GIVEN T
      C
        303 IF(KK.LE.2)GO TO 3030
            R(I)=R(I)-DELTA*2.0
            TCS3(K-1)=TCS3(K-2)
            GO TO 1303
       3030 R(I)=R(I)-DELTA
       1303 DELTA=DELTA/10.
            R(I)=R(I)+DELTA
            GO TO 304
      C
      C GROSS SEARCH ON T
      C
      C CALCULATION OF TOTAL COST
      C
        491 TCS2(J)=TCS3(K-1)
            P(J)=R(I)-DELTA
            IF(TCS2(J).LT.0.0)GO TO 1491
            GO TO 3134
       1491 TCS2(J)=99999999.00
            WRITE(6,4491)J,I
       4491 FORMAT('   VALUE FOR TOTAL COST WAS NEGATIVE FOR LOOP J=',I3,'  OF
          1 CASE #',I3)
            LINE=LINE+1
            IF(LINE.GT.45)GO TO 1436
            GO TO 309
      C
      C COMPARISON OF TOTAL COST TO DETERMINE CHANGE OF DIRECTION AND OPTIMAL T AND R
      C FOR A GIVEN ITEM
      C
       3134 IF(J.EQ.1)GO TO 309
            IF(TCS2(J).GT.TCS2(J-1).AND.ABS(DELTT-0.00274).LT.0.0001)GO TO 308
      C
      C FINE SEARCH ON T
      C
            IF(TCS2(J).GT.TCS2(J-1))GO TO 1308
        309 T(I)=T(I)+DELTT
            J=J+1
            GO TO 1302
       1308 IF(T(I).LE.0.03014)GO TO 1310
            T(I)=T(I)-DELTT*2.0
            DELTT=DELTT/10.
            T(I)=T(I)+DELTT
            J=J-1
            GO TO 1311
```

```
      1310 T(I)=T(I)-DELTT
           DELTT=0.00274
           T(I)=T(I)+DELTT
      1311 GO TO 302
C
C WRITE OUT OPTIMAL STOCHASTIC POLICY
C
       308 TCS1(I)=TCS2(J-1)
           RS(I)=P(J-1)
           TS(I)=T(I)-DELTT
           WRITE(6,3406)SIG1,MEA1,SIG2,MEA2,ENBO(I),EUYBO(I),TS(I),RS(I),TCS1
          1(I),I
      3406 FORMAT(1X,4(F9.2,1X),F11.2,1X,F8.2,2X,F10.6,1X,F10.2,1X,F11.2,4X,I
          13)
           LINE=LINE+1
           IF(LINE.GT.45)GO TO 1436
           GO TO 301
      1989 WRITE(6,1987)I
      1987 FORMAT(' THIS ITEM HAS NO SOLUTION IN THE DETERM. PART',47X,I3)
           WRITE(6,3406)SIG1,MEA1,SIG2,MEA2,ENBO(I),EUYBO(I),TS(I),RS(I),TCS1
          1(I),I
           LINE=LINE+2
           IF(LINE.GT.40)GO TO 1436
           GO TO 301
      1436 WRITE(6,704)
           LINE=0
       301 CONTINUE
C
C CALCULATION OF PERCENT DEVIATION FOR ALL ITEMS
C
           LINE=0
           WRITE(6,4111)
      4111 FORMAT(1H1,'      OPT T    OPT ORD     OPT T     OPT ORD       TOT CST
          1  TOT CST    TOT CST STC    PERCENT',/,'        DETRM   UP TO DET    STOC
          2H   UP TO STC      DETRM          STOCH       T,R DETRM   DEVIATION  ITE
          3M #',///)
           DO 311 I=1,N
           IF(OD(I).LT.0.0)GO TO 1986
C
C DETERMINE FROM OPTIMAL DETERMINISTIC POLICY THE DETERMINISTIC ESTIMATES OF T
C AND R
C
           T(I)=(OD(I)+SD(I))/DPY(I)
           IT(I)=(T(I)+0.00137)/0.00274
           T(I)=IT(I)*0.00274
           OD(I)=T(I)*DPY(I)
           R(I)=RD(I)+OD(I)
C
C DETERMINE CUMULATIVE DENSITY FUNCTION AND PROBABILITY DENSITY FUNCTION FOR
C FOUR STANDARD NORMAL RANDOM VARIABLES X1,X2,X3,X4
C
           SIG1=SQRT(DPY(I)*TAU(I))
           SIG2=SQRT(DPY(I)*(TAU(I)+T(I)))
           MEA1=DPY(I)*TAU(I)
           MEA2=DPY(I)*(TAU(I)+T(I))
           X1=(R(I)-MEA1)/SIG1
           X2=(R(I)-MEA2)/SIG2
```

```
      X3=(R(I)+MEA1)/SIG1
      X4=(R(I)+MEA2)/SIG2
C
C IF X GREATER THAN 5 CDF=0 AND IF X LESS THAN -5 CDF=1
C
      IF(X1.GE.5.)GO TO 513
      IF(X1.LE.-5.)GO TO 514
      TCDF1=RNORM(X1)
      CDF1=1-TCDF1
  521 IF(X2.GE.5.)GO TO 515
      IF(X2.LE.-5.)GO TO 516
      TCDF2=RNORM(X2)
      CDF2=1-TCDF2
  522 IF(X3.GE.5.)GO TO 517
      TCDF3=RNORM(X3)
      CDF3=1-TCDF3
  523 IF(X4.GE.5.)GO TO 518
      TCDF4=RNORM(X4)
      CDF4=1-TCDF4
C
C IF ABSOLUTE VALUE OF X GREATER THAN 13 PDF=0
C
  524 IF(ABS(X1).GE.13.)GO TO 519
      PDF1=1/(EXP(X1**2/2)*2.5066)
  525 IF(ABS(X2).GE.13.)GO TO 520
      PDF2=1/(EXP(X2**2/2)*2.5066)
  526 IF(ABS(CDF3-CDF4).LE.0.000001)GO TO 401
      GO TO 1526
  513 CDF1=0
      GO TO 521
  514 CDF1=1
      GO TO 521
  515 CDF2=0
      GO TO 522
  516 CDF2=1
      GO TO 522
  517 CDF3=0
      GO TO 523
  518 CDF4=0
      GO TO 524
  519 PDF1=0
      GO TO 525
  520 PDF2=0
      GO TO 526
C
C CALCULATION OF THE EXPECTED UNIT YEAR OF BACKORDER
C
 1526 Z1=(DPY(I)**2+2*DPY(I)**4*TAU(I)**2)/(4*DPY(I)**3)
      Z2=(DPY(I)-2.0*DPY(I)**2*TAU(I))*R(I)/(2.0*DPY(I)**2)
      Z3=R(I)**2/(2*DPY(I))
      Z4=SQRT(DPY(I))*SQRT(TAU(I)**3)
      Z5=SQRT(DPY(I)**3)*SQRT(TAU(I))/DPY(I)**2
      Z6=SIG1*R(I)/DPY(I)
      UTAU(I)=(Z1+Z2+Z3)*CDF1+0.5*(Z4-Z5-Z6)*PDF1-1/(4*DPY(I))*EXP(2*R(I
     1))*CDF3
      IF(UTAU(I).LT.0.0)UTAU(I)=0.0
      Z7=(DPY(I)**2+2*DPY(I)**4*(TAU(I)+T(I))**2)/(4*DPY(I)**3)
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      Z8=(DPY(I)-2*DPY(I)**2*(TAU(I)+T(I))*R(I))/(2*DPY(I)**2)
      Z9=R(I)**2/(2*DPY(I))
      Z10=SQRT(DPY(I))*SQRT((TAU(I)+T(I))**3)
      Z11=SQRT(DPY(I)**3)*SQRT(TAU(I)+T(I))/DPY(I)**2
      Z12=SIG2*R(I)/DPY(I)
      UTAUT(I)=(Z7+Z8+Z9)*CDF2+0.5*(Z10-Z11-Z12)*PDF2-EXP(2*R(I))*CDF4/(
     14*DPY(I))
      IF(UTAUT(I).LT.0.0)UTAUT(I)=0.0
      GO TO 412
  401 Z1=(DPY(I)**2+2*DPY(I)**4*TAU(I)**2)/(4*DPY(I)**3)
      Z2=(DPY(I)-2.0*DPY(I)**2*TAU(I))*R(I)/(2.0*DPY(I)**2)
      Z3=R(I)**2/(2*DPY(I))
      Z4=SQRT(DPY(I))*SQRT(TAU(I)**3)
      Z5=SQRT(DPY(I)**3)*SQRT(TAU(I))/DPY(I)**2
      Z6=SIG1*R(I)/DPY(I)
      UTAU(I)=(Z1+Z2+Z3)*CDF1+0.5*(Z4-Z5-Z6)*PDF1
      IF(UTAU(I).LT.0.0)UTAU(I)=0.0
      Z7=(DPY(I)**2+2*DPY(I)**4*(TAU(I)+T(I))**2)/(4*DPY(I)**3)
      Z8=(DPY(I)-2*DPY(I)**2*(TAU(I)+T(I))*R(I))/(2*DPY(I)**2)
      Z9=R(I)**2/(2*DPY(I))
      Z10=SQRT(DPY(I))*SQRT((TAU(I)+T(I))**3)
      Z11=SQRT(DPY(I)**3)*SQRT(TAU(I)+T(I))/DPY(I)**2
      Z12=SIG2*R(I)/DPY(I)
      UTAUT(I)=(Z7+Z8+Z9)*CDF2+0.5*(Z10-Z11-Z12)*PDF2
      IF(UTAUT(I).LT.0.0)UTAUT(I)=0.0
  412 EUYBO(I)=1/T(I)*(UTAUT(I)-UTAU(I))
      IF(EUYBO(I).LT.0.0)EUYBO(I)=0.0
C
C CALCULATION OF THE EXPECTED NUMBER OF BACKORDERS
C
      ENBO(I)=1/T(I)*(SIG2*PDF2-(R(I)-MEA2)*CDF2-SIG1*PDF1+(R(I)-MEA1)*C
     1DF1)
C
C CALCULATION OF TOTAL COST STOCHASTIC USING DETERMINISTIC POLICY
C
      TCSRD(I)=A(I)/T(I)+DI(I)*C(I)*(R(I)-MU(I)-DPY(I)*T(I)/2)+FPI(I)*EN
     1BO(I)+(DI(I)*C(I)+VPI(I))*EUYBO(I)
C
C CALCULATION OF PERCENT DEVIATION
C
      DEV(I)=(TCSRD(I)-TCS1(I))/TCS1(I)
C
C WRITE VALUES OF PERCENT DEVIATION
C
      WRITE(6,312)T(I),R(I),TS(I),RS(I),TCD(I),TCS1(I),TCSRD(I),DEV(I),I
  312 FORMAT(2(1X,F9.6,1X,F9.2),3(1X,F11.2),1X,F11.4,3X,I3)
      LINE=LINE+1
      IF(LINE.GT.45)GO TO 1437
      GO TO 311
 1986 WRITE(6,1987)I
      WRITE(6,312)T(I),R(I),TS(I),RS(I),TCD(I),TCS1(I),TCSRD(I),DEV(I),I
      LINE=LINE+2
      IF(LINE.GT.45)GO TO 1437
      GO TO 311
 1437 WRITE(6,4111)
      LINE=0
  311 CONTINUE
```

```
C
C CALCULATION OF PERCENT DEVIATION ADDING TO THE TOTAL STOCHASTIC COST THE
C ADDITIONAL COST FOR OPERATING THE SYSTEM WHEN THE STOCHASTIC POLICY IS USED TO
C OPERATE THE SYSTEM
C
      LINE=0
      COSP=OCSP
      DO 5000 K=1,M
      WRITE(6,5004)
 5004 FORMAT(1H1,'      OPT T    OPT ORD     OPT T     OPT ORD       TOT CST  T
     1OT CST STC  TOT CST STC    OPERATING    PERCENT',/,'       DETRM  UP T
     2O DET    STOCH    UP TO STC     DETERM    WITH OPR CT    T,R DETRM
     3 COST       DEVIATION  ITEM #',///)
      OCSP=COSP**K
      DO 5001 I=1,N
      IF(RD(I).LE.0.0)GO TO 5002
      TCS4(I)=TCS1(I)+OCSP
      DEV(I)=(TCSRD(I)-TCS4(I))/TCS4(I)
      WRITE(6,5006)T(I),R(I),TS(I),RS(I),TCD(I),TCS4(I),TCSRD(I),OCSP,DE
     1V(I),I
 5006 FORMAT(2(1X,F9.6,1X,F9.2),4(1X,F11.2),1X,F11.4,3X,I3)
      LINE=LINE+1
      IF (LINE.GT.45)GO TO 1237
      GO TO 5001
 5002 WRITE(6,5007)I
 5007 FORMAT(' THIS ITEM HAS NO SOLUTION IN THE DETERM. PART',59X,I3)
      LINE=LINE+1
      IF (LINE.GT.45)GO TO 1237
      GO TO 5001
 1237 WRITE(6,5004)
      LINE=0
 5001 CONTINUE
      LINE=0
 5000 CONTINUE
      CALL EXIT
      END

END OF COMPILATION:        NO  DIAGNOSTICS.
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APPENDIX IV

SAMPLE PRINTOUT FOR Q PROGRAM

AVG DEM PER YEAR	ORDING COST	CARR CHRG	UNIT COST	FIX BCK ORD CST	VAR BCK ORD CST	LEAD TIME	OPT Q WILS	TOT CST WILSON	LEAD TIM DEMAND	OPT Q DET	NUM BCK ORDER	REORD POINT	TOT CST DET	
3500.0	450.00	.20	300.00	1.00	1000.00	.10	229.13	13747.73	350.0	235.47	10.03	339.97	13526.57	1
3500.0	450.00	.20	300.00	1.00	1000.00	.03	229.13	13747.73	105.0	235.47	10.03	94.97	13526.57	2
3500.0	450.00	.20	300.00	1.00	350.00	.10	229.13	13747.73	350.0	246.81	27.58	322.42	13153.84	3
3500.0	450.00	.20	300.00	1.00	350.00	.03	229.13	13747.73	105.0	246.81	27.58	77.42	13153.84	4
3500.0	450.00	.20	300.00	.10	1000.00	.10	229.13	13747.73	350.0	235.90	13.02	336.98	13372.54	5
3500.0	450.00	.20	300.00	.10	1000.00	.03	229.13	13747.73	105.0	235.90	13.02	91.98	13372.54	6
3500.0	450.00	.20	300.00	.10	350.00	.10	229.13	13747.73	350.0	247.98	35.44	314.56	12752.64	7
3500.0	450.00	.20	300.00	.10	350.00	.03	229.13	13747.73	105.0	247.98	35.44	69.56	12752.64	8
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 9														
3500.0	450.00	.20	8.00	1.00	1000.00	.10	1403.12	2244.99	350.0	1403.12	.00	350.00	2244.99	9
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 10														
3500.0	450.00	.20	8.00	1.00	1000.00	.03	1403.12	2244.99	105.0	1403.12	.00	105.00	2244.99	10
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 11														
3500.0	450.00	.20	8.00	1.00	350.00	.10	1403.12	2244.99	350.0	1403.12	.00	350.00	2244.99	11
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 12														
3500.0	450.00	.20	8.00	1.00	350.00	.03	1403.12	2244.99	105.0	1403.12	.00	105.00	2244.99	12
3500.0	450.00	.20	8.00	.10	1000.00	.10	1403.12	2244.99	350.0	1404.22	1.89	348.11	2243.72	13
3500.0	450.00	.20	8.00	.10	1000.00	.03	1403.12	2244.99	105.0	1404.22	1.89	103.11	2243.72	14
3500.0	450.00	.20	8.00	.10	350.00	.10	1403.12	2244.99	350.0	1406.25	5.40	344.60	2241.35	15
3500.0	450.00	.20	8.00	.10	350.00	.03	1403.12	2244.99	105.0	1406.25	5.40	99.60	2241.35	16
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 17														
3500.0	10.00	.20	300.00	1.00	1000.00	.10	34.16	2049.39	350.0	34.16	.00	350.00	2049.39	17
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 18														
3500.0	10.00	.20	300.00	1.00	1000.00	.03	34.16	2049.39	105.0	34.16	.00	105.00	2049.39	18
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 19														
3500.0	10.00	.20	300.00	1.00	350.00	.10	34.16	2049.39	350.0	34.16	.00	350.00	2049.39	19
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 20														
3500.0	10.00	.20	300.00	1.00	350.00	.03	34.16	2049.39	105.0	34.16	.00	105.00	2049.39	20
3500.0	10.00	.20	300.00	.10	1000.00	.10	34.16	2049.39	350.0	35.14	1.66	348.34	2008.71	21
3500.0	10.00	.20	300.00	.10	1000.00	.03	34.16	2049.39	105.0	35.14	1.66	103.34	2008.71	22
3500.0	10.00	.20	300.00	.10	350.00	.10	34.16	2049.39	350.0	36.89	4.54	345.46	1940.68	23
3500.0	10.00	.20	300.00	.10	350.00	.03	34.16	2049.39	105.0	36.89	4.54	100.46	1940.68	24
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 25														
3500.0	10.00	.20	8.00	1.00	1000.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	25
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 26														
3500.0	10.00	.20	8.00	1.00	1000.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	26
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 27														
3500.0	10.00	.20	8.00	1.00	350.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	27
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 28														
3500.0	10.00	.20	8.00	1.00	350.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	28
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 29														
3500.0	10.00	.20	8.00	.10	1000.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	29
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 30														
3500.0	10.00	.20	8.00	.10	1000.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	30
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 31														
3500.0	10.00	.20	8.00	.10	350.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	31

AVG DEM PER YEAR	ORDING COST	CARR CHRG	UNIT COST	FIX BCK ORD CST	VAR BCK ORD CST	LEAD TIME	OPT Q WILS	TOT CST WILSON	LEAD TIM DEMAND	OPT Q DET	NUM BCK ORDER	REORD POINT	TOT CST DET	
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 32														
3500.0	10.00	.20	8.00	.10	350.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.65	32
400.0	450.00	.20	300.00	1.00	1000.00	.10	77.46	4647.58	40.0	79.73	4.14	35.86	4535.82	33
400.0	450.00	.20	300.00	1.00	1000.00	.03	77.46	4647.58	12.0	79.73	4.14	7.86	4535.82	34
400.0	450.00	.20	300.00	1.00	350.00	.10	77.46	4647.58	40.0	83.79	11.29	28.71	4350.29	35
400.0	450.00	.20	300.00	1.00	350.00	.03	77.46	4647.58	12.0	83.79	11.29	.71	4350.28	36
400.0	450.00	.20	300.00	.10	1000.00	.10	77.46	4647.58	40.0	79.75	4.48	35.52	4516.38	37
400.0	450.00	.20	300.00	.10	1000.00	.03	77.46	4647.58	12.0	79.75	4.48	7.52	4516.39	38
400.0	450.00	.20	300.00	.10	350.00	.10	77.46	4647.58	40.0	83.84	12.17	27.83	4299.97	39
400.0	450.00	.20	300.00	.10	350.00	.03	77.46	4647.58	12.0	83.84	12.17	-.17	4299.99	40
400.0	450.00	.20	8.00	1.00	1000.00	.10	474.34	758.95	40.0	474.62	.36	39.64	759.61	41
400.0	450.00	.20	8.00	1.00	1000.00	.03	474.34	758.95	12.0	474.62	.36	11.64	759.81	42
400.0	450.00	.20	8.00	1.00	350.00	.10	474.34	758.95	40.0	475.12	1.02	38.98	758.56	43
400.0	450.00	.20	8.00	1.00	350.00	.03	474.34	758.95	12.0	475.12	1.02	10.98	758.56	44
400.0	450.00	.20	8.00	.10	1000.00	.10	474.34	758.95	40.0	474.72	.72	39.28	755.40	45
400.0	450.00	.20	8.00	.10	1000.00	.03	474.34	758.95	12.0	474.72	.72	11.28	755.43	46
400.0	450.00	.20	8.00	.10	350.00	.10	474.34	758.95	40.0	475.42	2.05	37.95	757.40	47
400.0	450.00	.20	8.00	.10	350.00	.03	474.34	758.95	12.0	475.42	2.05	9.95	757.40	48
400.0	10.00	.20	300.00	1.00	1000.00	.10	11.55	692.82	40.0	11.78	.29	39.71	689.19	49
400.0	10.00	.20	300.00	1.00	1000.00	.03	11.55	692.82	12.0	11.78	.29	11.71	689.19	50
400.0	10.00	.20	300.00	1.00	350.00	.10	11.55	692.82	40.0	12.19	.81	39.19	682.85	51
400.0	10.00	.20	300.00	1.00	350.00	.03	11.55	692.82	12.0	12.19	.81	11.19	682.85	52
400.0	10.00	.20	300.00	.10	1000.00	.10	11.55	692.82	40.0	11.89	.64	39.36	675.13	53
400.0	10.00	.20	300.00	.10	1000.00	.03	11.55	692.82	12.0	11.89	.64	11.36	675.13	54
400.0	10.00	.20	300.00	.10	350.00	.10	11.55	692.82	40.0	12.49	1.73	38.27	645.52	55
400.0	10.00	.20	300.00	.10	350.00	.03	11.55	692.82	12.0	12.49	1.73	10.27	645.82	56
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 57														
400.0	10.00	.20	8.00	1.00	1000.00	.10	70.71	113.14	40.0	70.71	.00	40.00	113.14	57
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 58														
400.0	10.00	.20	8.00	1.00	1000.00	.03	70.71	113.14	12.0	70.71	.00	12.00	113.14	58
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 59														
400.0	10.00	.20	8.00	1.00	350.00	.10	70.71	113.14	40.0	70.71	.00	40.00	113.14	59
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 60														
400.0	10.00	.20	8.00	1.00	350.00	.03	70.71	113.14	12.0	70.71	.00	12.00	113.14	60
400.0	10.00	.20	8.00	.10	1000.00	.10	70.71	113.14	40.0	70.76	.07	39.93	113.10	61
400.0	10.00	.20	8.00	.10	1000.00	.03	70.71	113.14	12.0	70.76	.07	11.93	113.10	62
400.0	10.00	.20	8.00	.10	350.00	.10	70.71	113.14	40.0	70.85	.21	39.79	113.03	63
400.0	10.00	.20	8.00	.10	350.00	.03	70.71	113.14	12.0	70.85	.21	11.79	113.03	64

STDV OF DEM DTR LEAD TM	MEAN OF DEM DTR LEAD TM	OPT Q STOC	REORD POINT	TOT CST STOC	ITEM #
12.5	350.0	240.33	340.59	13855.51	1
12.5	105.0	240.33	95.59	13855.51	2
12.5	350.0	249.39	322.00	13283.37	3
12.5	105.0	249.39	77.00	13283.37	4
12.5	350.0	240.52	337.75	13696.18	5
12.5	105.0	240.52	92.75	13696.18	6
12.5	350.0	250.47	314.21	12881.13	7
12.5	105.0	250.47	69.21	12881.13	8
12.5	350.0	1409.50	359.98	2271.17	9
12.5	105.0	1409.50	114.98	2271.17	10
12.5	350.0	1410.85	354.83	2265.09	11
12.5	105.0	1410.85	109.83	2265.09	12
12.5	350.0	1409.63	357.34	2267.15	13
12.5	105.0	1409.63	112.34	2267.15	14
12.5	350.0	1411.55	348.33	2255.80	15
12.5	105.0	1411.55	103.33	2255.80	16
12.5	350.0	41.18	359.57	3044.96	17
12.5	105.0	41.18	114.57	3044.96	18
12.5	350.0	42.80	354.28	2824.67	19
12.5	105.0	42.80	109.28	2824.67	20
12.5	350.0	41.32	357.02	2900.45	21
12.5	105.0	41.32	112.02	2900.45	22
12.5	350.0	43.56	348.27	2510.06	23
12.5	105.0	43.56	103.27	2510.06	24
12.5	350.0	213.96	371.89	377.36	25
12.5	105.0	213.96	126.89	377.36	26
12.5	350.0	214.45	369.09	373.66	27
12.5	105.0	214.45	124.09	373.66	28
12.5	350.0	214.05	369.47	373.62	29
12.5	105.0	214.05	124.47	373.62	30
12.5	350.0	214.81	363.86	365.87	31
12.5	105.0	214.81	118.86	365.87	32
12.5	40.0	85.61	40.66	5176.49	33
12.5	12.0	85.61	12.66	5176.49	34
12.5	40.0	89.12	29.06	4691.17	35
12.5	12.0	89.12	1.06	4691.17	36
12.5	40.0	85.62	40.32	5156.53	37
12.5	12.0	85.62	12.32	5156.53	38
12.5	40.0	89.14	28.20	4640.26	39
12.5	12.0	89.14	.20	4640.26	40
12.5	40.0	479.75	54.85	791.36	41
12.5	12.0	479.75	26.85	791.36	42
12.5	40.0	480.82	48.30	782.59	43
12.5	12.0	480.82	20.30	782.59	44
12.5	40.0	479.75	54.50	790.81	45
12.5	12.0	479.75	26.50	790.81	46
12.5	40.0	480.84	47.33	781.07	47
12.5	12.0	480.84	19.33	781.07	48
12.5	40.0	18.50	52.84	1880.46	49
12.5	12.0	18.50	24.84	1880.46	50
12.5	40.0	20.31	45.68	1559.40	51
12.5	12.0	20.31	17.68	1559.40	52
12.5	40.0	18.51	52.50	1860.70	53
12.5	12.0	10.51	24.50	1860.70	54
12.5	40.0	20.34	44.83	1509.89	55
12.5	12.0	20.33	16.84	1509.89	56
12.5	40.0	75.13	64.81	159.91	57
12.5	12.0	75.13	36.81	159.91	58
12.5	40.0	75.69	60.14	153.32	59
12.5	12.0	75.69	32.14	153.32	60
12.5	40.0	75.13	64.47	159.36	61
12.5	12.0	75.13	36.47	159.36	62
12.5	40.0	75.71	59.20	151.85	63
12.5	12.0	75.71	31.20	151.85	64

OPT Q DETRM	REORD PT DETRM	OPT Q STOCH	REORD PT STOCH	TOT CST DETRM	TOT CST STOCH	TOT CST STC Q,R DETRM	PERCENT DEVIATION	ITEM #
235.47	339,97	240.33	340.59	13526.57	13855.51	13860.00	.0003	1
235.47	94,97	240.33	95.59	13526.57	13855.51	13860.00	.0003	2
246.81	322,42	249.39	322.00	13153.84	13283.37	13284.07	.0001	3
246.81	77,42	249.39	77.00	13153.84	13283.37	13284.07	.0001	4
235.90	336,98	240.52	337.75	13372.54	13696.18	13700.92	.0003	5
235.90	91,98	240.52	92.75	13372.54	13696.18	13700.92	.0003	6
247.98	314,56	250.47	314.21	12752.64	12881.13	12881.78	.0001	7
247.98	69,56	250.47	69.21	12752.64	12881.13	12881.78	.0001	8
1403.12	350,00	1409.50	359.98	2244.99	2271.17	2285.32	.0062	9
1403.12	105,00	1409.50	114.98	2244.99	2271.17	2285.32	.0062	10
1403.12	350,00	1410.85	354.83	2244.99	2265.09	2267.22	.0009	11
1403.12	105,00	1410.85	109.83	2244.99	2265.09	2267.22	.0009	12
1404.22	348,11	1409.63	357.34	2243.72	2267.15	2278.72	.0051	13
1404.22	103,11	1409.63	112.34	2243.72	2267.15	2278.72	.0051	14
1406.25	344,60	1411.55	348.33	2241.35	2255.80	2256.92	.0005	15
1406.25	99,60	1411.55	103.33	2241.35	2255.80	2256.92	.0005	16
34.16	350,00	41.18	359.57	2049.39	3044.96	3772.64	.2390	17
34.16	105,00	41.18	114.57	2049.39	3044.96	3772.64	.2390	18
34.16	350,00	42.80	354.28	2049.39	2824.67	3029.28	.0724	19
34.16	105,00	42.80	109.28	2049.39	2824.67	3029.28	.0724	20
35.14	348,34	41.32	357.02	2008.71	2900.45	3458.49	.1924	21
35.14	103,34	41.32	112.02	2008.71	2900.45	3458.49	.1924	22
36.69	345,46	43.56	348.27	1940.68	2510.06	2603.70	.0373	23
36.89	100,46	43.56	103.27	1940.68	2510.06	2603.70	.0373	24
209.17	350,00	213.96	371.89	334.66	377.36	605.16	.6037	25
209.17	105,00	213.96	126.89	334.66	377.36	605.16	.6037	26
209.17	350,00	214.45	369.09	334.66	373.66	483.77	.2947	27
209.17	105,00	214.45	124.09	334.66	373.66	483.77	.2947	28
209.17	350.00	214.05	369.47	334.66	373.62	530.06	.4187	29
209.17	105.00	214.05	124.47	334.66	373.62	530.06	.4187	30
209.17	350,00	214.81	363.86	334.66	365.87	408.67	.1170	31
209.17	105,00	214.81	118.86	334.66	365.87	408.67	.1170	32
79.73	35,86	85.61	40.66	4535.82	5176.49	5293.45	.0226	33
79.73	7,86	85.61	12.66	4535.82	5176.49	5293.45	.0226	34
83.79	28,71	89.12	29.06	4350.28	4691.17	4702.93	.0025	35
83.79	.71	89.12	1.06	4350.28	4691.17	4702.93	.0025	36
79.75	35,52	85.62	40.32	4516.38	5156.53	5273.55	.0227	37
79.75	7,52	85.62	12.32	4516.38	5156.53	5273.55	.0227	38
83.84	27,83	89.14	28.20	4299.90	4640.26	4651.99	.0025	39
83.84	-,17	89.14	.20	4299.90	4640.26	4651.99	.0025	40
474.62	39,64	479.75	54.85	758.81	791.36	849.01	.0728	41
474.62	11,64	479.75	26.85	758.81	791.36	849.01	.0728	42
475.12	38,98	480.82	48.30	758.56	782.59	794.84	.0157	43
475.12	10,98	480.82	20.30	758.56	782.59	794.84	.0157	44
474.72	39,28	479.75	54.50	758.40	790.81	848.50	.0730	45
474.72	11,28	479.75	26.50	758.40	790.81	848.50	.0730	46

OPT Q DETRM	REORD PT DETRM	OPT Q STOCH	REORD PT STOCH	TOT CST DETRM	TOT CST STOCH	TOT CST STC Q,R DETRM	PERCENT DEVIATION	ITEM #
475.42	37,95	480.84	47.33	757.40	781.07	793.44	.0158	47
475.42	9,95	480.84	19.33	757.40	781.07	793.44	.0158	48
11.78	39,71	18.50	52.84	689.19	1880.46	4497,91	1.3919	49
11.78	11,71	18.50	24.84	689.19	1880.46	4497,91	1.3919	50
12.19	39,19	20.31	45.68	682.85	1559.40	2277.69	.4606	51
12.19	11,19	20.31	17.68	682.85	1559.40	2277.69	.4606	52
11.89	39,36	18.51	52.50	675.13	1860.70	4447.67	1.3903	53
11.89	11,36	18.51	24.50	675.13	1860.70	4447.67	1.3903	54
12.49	38,27	20.34	44.83	645.82	1509.89	2200.54	.4574	55
12.49	10,27	20.33	16.84	645.82	1509.89	2200.54	.4574	56
70.71	40,00	75.13	64.81	113.14	159.91	694.66	3.3440	57
70.71	12,00	75.13	36.81	113.14	159.91	694.66	3.3440	58
70.71	40,00	75.69	60.14	113.14	153.32	335.58	1.1887	59
70.71	12,00	75.69	32.14	113.14	153.32	335.58	1.1887	60
70.76	39,93	76.13	64.47	113.10	159.36	673.96	3.2291	61
70.76	11,93	75.13	36.47	113.10	159.36	673.96	3.2291	62
70.85	39,79	75.71	59.20	113.03	151.85	314.74	1.0727	63
70.85	11,79	75.71	31.20	113.03	151.85	314.74	1.0727	64

APPENDIX V

SAMPLE PRINTOUT FOR R PROGRAM

AVG DEM PER YEAR	ORDING COST	CARR CHRG	UNIT COST	FIX BCK ORD CST	VAR BCK ORD CST	LEAD TIME	OPT Q WILS	TOT CST WILSON	LEAD TIM DEMAND	OPT Q DET	NUM BCK ORDER	REORD POINT	TOT CST DET	
3500.0	450.00	.20	300.00	1.00	1000.00	.10	229.13	13747.73	350.0	235.47	10.03	339.97	13526.57	1
3500.0	450.00	.20	300.00	1.00	1000.00	.03	229.13	13747.73	105.0	235.47	10.03	94.97	13526.57	2
3500.0	450.00	.20	300.00	1.00	350.00	.10	229.13	13747.73	350.0	246.81	27.58	322.42	13153.84	3
3500.0	450.00	.20	300.00	1.00	350.00	.03	229.13	13747.73	105.0	246.81	27.58	77.42	13153.84	4
3500.0	450.00	.20	300.00	.10	1000.00	.10	229.13	13747.73	350.0	235.90	13.02	336.98	13372.54	5
3500.0	450.00	.20	300.00	.10	1000.00	.03	229.13	13747.73	105.0	235.90	13.02	91.98	13372.54	6
3500.0	450.00	.20	300.00	.10	350.00	.10	229.13	13747.73	350.0	247.98	35.44	314.56	12752.64	7
3500.0	450.00	.20	300.00	.10	350.00	.03	229.13	13747.73	105.0	247.98	35.44	69.56	12752.64	8
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 9														
3500.0	450.00	.20	8.00	1.00	1000.00	.10	1403.12	2244.99	350.0	1403.12	.00	350.00	2244.99	9
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 10														
3500.0	450.00	.20	8.00	1.00	1000.00	.03	1403.12	2244.99	105.0	1403.12	.00	105.00	2244.99	10
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 11														
3500.0	450.00	.20	8.00	1.00	350.00	.10	1403.12	2244.99	350.0	1403.12	.00	350.00	2244.99	11
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 12														
3500.0	450.00	.20	8.00	1.00	350.00	.03	1403.12	2244.99	105.0	1403.12	.00	105.00	2244.99	12
3500.0	450.00	.20	8.00	.10	1000.00	.10	1403.12	2244.99	350.0	1404.22	1.89	348.11	2243.72	13
3500.0	450.00	.20	8.00	.10	1000.00	.03	1403.12	2244.99	105.0	1404.22	1.89	103.11	2243.72	14
3500.0	450.00	.20	8.00	.10	350.00	.10	1403.12	2244.99	350.0	1406.25	5.40	344.60	2241.35	15
3500.0	450.00	.20	8.00	.10	350.00	.03	1403.12	2244.99	105.0	1406.25	5.40	99.60	2241.35	16
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 17														
3500.0	10.00	.20	300.00	1.00	1000.00	.10	34.16	2049.39	350.0	34.16	.00	350.00	2049.39	17
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 18														
3500.0	10.00	.20	300.00	1.00	1000.00	.03	34.16	2049.39	105.0	34.16	.00	105.00	2049.39	18
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 19														
3500.0	10.00	.20	300.00	1.00	350.00	.10	34.16	2049.39	350.0	34.16	.00	350.00	2049.39	19
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 20														
3500.0	10.00	.20	300.00	1.00	350.00	.03	34.16	2049.39	105.0	34.16	.00	105.00	2049.39	20
3500.0	10.00	.20	300.00	.10	1000.00	.10	34.16	2049.39	350.0	35.14	1.66	348.34	2008.71	21
3500.0	10.00	.20	300.00	.10	1000.00	.03	34.16	2049.39	105.0	35.14	1.66	103.34	2008.71	22
3500.0	10.00	.20	300.00	.10	350.00	.10	34.16	2049.39	350.0	36.89	4.54	345.46	1940.68	23
3500.0	10.00	.20	300.00	.10	350.00	.03	34.16	2049.39	105.0	36.89	4.54	100.46	1940.68	24
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 25														
3500.0	10.00	.20	8.00	1.00	1000.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	25
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 26														
3500.0	10.00	.20	8.00	1.00	1000.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	26
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 27														
3500.0	10.00	.20	8.00	1.00	350.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	27
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 28														
3500.0	10.00	.20	8.00	1.00	350.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	28
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 29														
3500.0	10.00	.20	8.00	.10	1000.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	29
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 30														
3500.0	10.00	.20	8.00	.10	1000.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.66	30
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 31														
3500.0	10.00	.20	8.00	.10	350.00	.10	209.17	334.66	350.0	209.17	.00	350.00	334.66	31

AVG DEM PER YEAR	ORDING COST	CARR CHRG	UNIT COST	FIX BCK ORD CST	VAR BCK ORD CST	LEAD TIME	OPT Q WILS	TOT CST WILSON	LEAD TIM DEMAND	OPT Q DET	NUM BCK ORDER	REORD POINT	TOT CST DET	
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 32														
3500.0	10.00	.20	8.00	.10	350.00	.03	209.17	334.66	105.0	209.17	.00	105.00	334.65	32
400.0	450.00	.20	300.00	1.00	1000.00	.10	77.46	4647.58	40.0	79.73	4.14	35.86	4535.82	33
400.0	450.00	.20	300.00	1.00	1000.00	.03	77.46	4647.58	12.0	79.73	4.14	7.86	4535.82	34
400.0	450.00	.20	300.00	1.00	350.00	.10	77.46	4647.58	40.0	83.79	11.29	28.71	4350.28	35
400.0	450.00	.20	300.00	1.00	350.00	.03	77.46	4647.58	12.0	83.79	11.29	.71	4350.28	36
400.0	450.00	.20	300.00	.10	1000.00	.10	77.46	4647.58	40.0	79.75	4.48	35.52	4516.35	37
400.0	450.00	.20	300.00	.10	1000.00	.03	77.46	4647.58	12.0	79.75	4.48	7.52	4516.38	38
400.0	450.00	.20	300.00	.10	350.00	.10	77.46	4647.58	40.0	83.84	12.17	27.83	4299.90	39
400.0	450.00	.20	300.00	.10	350.00	.03	77.46	4647.58	12.0	83.84	12.17	-.17	4299.90	40
400.0	450.00	.20	8.00	1.00	1000.00	.10	474.34	759.95	40.0	474.62	.36	39.64	759.81	41
400.0	450.00	.20	8.00	1.00	1000.00	.03	474.34	758.95	12.0	474.62	.36	11.64	758.81	42
400.0	450.00	.20	8.00	1.00	350.00	.10	474.34	758.95	40.0	475.12	1.02	38.98	758.55	43
400.0	450.00	.20	8.00	1.00	350.00	.03	474.34	758.95	12.0	475.12	1.02	10.98	758.56	44
400.0	450.00	.20	8.00	.10	1000.00	.10	474.34	758.95	40.0	474.72	.72	39.28	755.40	45
400.0	450.00	.20	8.00	.10	1000.00	.03	474.34	758.95	12.0	474.72	.72	11.28	755.40	46
400.0	450.00	.20	8.00	.10	350.00	.10	474.34	758.95	40.0	475.42	2.05	37.95	757.45	47
400.0	450.00	.20	8.00	.10	350.00	.03	474.34	758.95	12.0	475.42	2.05	9.95	757.40	48
400.0	10.00	.20	300.00	1.00	1000.00	.10	11.55	692.82	40.0	11.78	.29	39.71	689.19	49
400.0	10.00	.20	300.00	1.00	1000.00	.03	11.55	692.82	12.0	11.78	.29	11.71	689.19	50
400.0	10.00	.20	300.00	1.00	350.00	.10	11.55	692.82	40.0	12.19	.81	39.19	682.85	51
400.0	10.00	.20	300.00	1.00	350.00	.03	11.55	692.82	12.0	12.19	.81	11.19	682.85	52
400.0	10.00	.20	300.00	.10	1000.00	.10	11.55	692.82	40.0	11.89	.64	39.36	675.13	53
400.0	10.00	.20	300.00	.10	1000.00	.03	11.55	692.82	12.0	11.89	.64	11.36	675.13	54
400.0	10.00	.20	300.00	.10	350.00	.10	11.55	692.82	40.0	12.49	1.73	38.27	645.82	55
400.0	10.00	.20	300.00	.10	350.00	.03	11.55	692.82	12.0	12.49	1.73	10.27	645.82	56
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 57														
400.0	10.00	.20	8.00	1.00	1000.00	.10	70.71	113.14	40.0	70.71	.00	40.00	113.14	57
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 58														
400.0	10.00	.20	8.00	1.00	1000.00	.03	70.71	113.14	12.0	70.71	.00	12.00	113.14	58
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 59														
400.0	10.00	.20	8.00	1.00	350.00	.10	70.71	113.14	40.0	70.71	.00	40.00	113.14	59
THE NUMBER OF BACKORDERS SD,IS NEGATIVE SO SD=0 AND QD=QWILSON FOR ITEM # 60														
400.0	10.00	.20	8.00	1.00	350.00	.03	70.71	113.14	12.0	70.71	.00	12.00	113.14	60
400.0	10.00	.20	8.00	.10	1000.00	.10	70.71	113.14	40.0	70.76	.07	39.93	113.10	61
400.0	10.00	.20	8.00	.10	1000.00	.03	70.71	113.14	12.0	70.76	.07	11.93	113.10	62
400.0	10.00	.20	8.00	.10	350.00	.10	70.71	113.14	40.0	70.85	.21	39.79	113.03	63
400.0	10.00	.20	8.00	.10	350.00	.03	70.71	113.14	12.0	70.85	.21	11.79	113.03	64

STDV DEM DIST TAU	MEAN DEM DIST TAU	STDV DEM DIS TAU+T	MEAN DEM DIS TAU+T	EXP NUM BACKORD	EXP UNT-YR BACKOR	OPT CYLE TIME STCH	ORD UP TO LEVEL STC	TOT CST STOCH	ITEM #
18.71	350.00	24.48	599.34	226.40	.33	.068500	579.40	13713.03	1
10.25	105.00	18.82	354.34	205.62	.33	.068500	333.70	13641.96	2
18.71	350.00	24.68	608.93	436.05	1.93	.071240	569.60	13205.66	3
10.25	105.00	19.08	363.93	414.84	1.79	.071240	325.30	13173.71	4
18.71	350.00	24.48	599.34	251.58	.47	.068500	576.90	13495.39	5
10.25	105.00	18.82	354.34	231.99	.46	.068500	331.20	13443.71	6
18.71	350.00	24.68	608.93	519.15	2.80	.071240	562.90	12774.96	7
10.25	105.00	19.08	363.93	507.05	2.70	.071240	318.40	12759.40	8
18.71	350.00	42.18	1778.91	42.22	.00	.405520	1768.40	2286.20	9
10.25	105.00	39.17	1533.91	39.27	.00	.405520	1523.40	2283.22	10
18.71	350.00	42.18	1778.91	42.22	.00	.405520	1768.40	2286.20	11
10.25	105.00	39.17	1533.91	39.27	.00	.405520	1523.40	2283.22	12
18.71	350.00	42.06	1769.32	42.40	.00	.402780	1758.80	2247.81	13
10.25	105.00	39.04	1524.32	39.43	.00	.402780	1513.80	2247.54	14
18.71	350.00	42.06	1769.32	42.40	.00	.402780	1758.80	2247.80	15
10.25	105.00	39.04	1524.32	39.43	.00	.402780	1513.80	2247.52	16
18.71	350.00	20.19	407.54	512.34	.00	.013700	397.10	2727.29	17
10.25	105.00	12.37	152.95	384.36	.00	.010960	142.60	2488.96	18
18.71	350.00	20.19	407.54	515.47	.00	.013700	397.00	2727.08	19
10.25	105.00	12.37	152.95	388.17	.00	.010960	142.60	2488.96	20
18.71	350.00	19.71	388.36	747.45	.00	.008220	377.00	2071.47	21
10.25	105.00	12.37	152.95	388.17	.00	.010960	142.60	2064.74	22
18.71	350.00	19.95	397.95	748.86	.39	.010960	385.80	2067.66	23
10.25	105.00	12.37	152.95	618.18	.49	.010960	139.10	2018.53	24
18.71	350.00	23.89	570.57	17.92	.00	.060280	591.80	401.83	25
10.25	105.00	18.04	325.57	13.55	.00	.060280	339.10	395.08	26
18.71	350.00	23.89	570.57	17.92	.00	.060280	591.80	401.83	27
10.25	105.00	18.04	325.57	13.55	.00	.060280	339.10	395.08	28
18.71	350.00	24.09	580.16	151.98	.00	.063020	569.70	349.57	29
10.25	105.00	18.04	325.57	119.62	.00	.060280	315.20	345.85	30
18.71	350.00	24.09	580.16	151.98	.00	.063020	569.70	349.57	31
10.25	105.00	18.04	325.57	119.62	.00	.060280	315.20	345.85	32
6.32	40.00	11.00	121.10	31.07	.11	.200020	116.60	4597.42	33
3.46	12.00	9.65	93.10	29.50	.12	.200020	88.30	4596.76	34
6.32	40.00	11.20	125.49	59.51	.88	.210980	112.60	4375.53	35
3.46	12.00	9.87	97.49	57.78	.85	.210980	84.60	4367.76	36
6.32	40.00	10.95	120.01	32.04	.12	.197280	115.30	4568.67	37
3.46	12.00	9.65	93.10	30.47	.14	.200020	88.00	4559.51	38
6.32	40.00	11.20	125.49	62.74	.99	.210980	111.80	4320.17	39
3.46	12.00	9.87	97.49	61.12	.96	.210980	83.90	4313.78	40
6.32	40.00	22.73	516.76	7.93	.00	1.189160	514.80	765.55	41
3.46	12.00	22.11	488.76	7.72	.00	1.189160	486.80	765.34	42
6.32	40.00	22.73	516.76	7.93	.00	1.189160	514.80	765.55	43
3.46	12.00	22.11	488.76	7.72	.00	1.189160	486.80	765.34	44
6.32	40.00	22.71	515.66	7.94	.00	1.186420	513.70	758.36	45
3.46	12.00	22.08	487.66	7.73	.00	1.186420	485.70	758.34	46

STDV DEM DIST TAU	MEAN DEM DIST TAU	STDV DEM DIS TAU+T	MEAN DEM DIS TAU+T	EXP NUM BACKORD	EXP UNT-YR BACKOR	OPT CYLE TIME STCH	ORD UP TO LEVEL STC	TOT CST STOCH	ITEM #
6.32	40.00	22.68	514.57	7.96	.00	1.183680	512.60	758.36	47
3.46	12.00	22.08	487.66	7.73	.00	1.186420	485.70	758.34	48
6.32	40.00	7.37	54.25	89.46	.00	.032880	52.50	756.56	49
3.46	12.00	5.12	26.25	63.89	.00	.032880	24.60	738.83	50
6.32	40.00	7.29	53.15	96.99	.00	.030140	51.30	754.83	51
3.46	12.00	5.12	26.25	66.93	.01	.032880	24.60	736.59	52
6.32	40.00	7.21	52.06	103.80	.00	.027400	50.20	659.48	53
3.46	12.00	5.02	25.15	67.97	.00	.030140	23.50	672.94	54
6.32	40.00	7.21	52.06	103.80	.00	.027400	50.10	658.18	55
3.46	12.00	5.02	25.15	81.77	.09	.030140	23.00	666.33	56
6.32	40.00	10.70	114.53	10.68	.00	.183580	119.20	133.20	57
3.46	12.00	9.18	84.34	9.10	.00	.178100	88.40	130.53	58
6.32	40.00	10.70	114.53	10.68	.00	.183580	119.20	133.20	59
3.46	12.00	9.18	84.34	9.10	.00	.178100	88.40	130.53	60
6.32	40.00	10.60	112.34	25.18	.00	.178100	110.50	114.53	61
3.46	12.00	9.18	84.34	21.78	.00	.178100	82.60	114.34	62
6.32	40.00	10.60	112.34	25.18	.00	.178100	110.50	114.53	63
3.46	12.00	9.18	84.34	21.78	.00	.178100	82.60	114.34	64

OPT T DETRM	OPT ORD UP TO DET	OPT T STOCH	OPT ORD UP TO STC	TOT CST DETRM	TOT CST STOCH	TOT CST STC T/R DETRM	PERCENT DEVIATION	ITEM #
.071240	589.31	.068500	579.40	13526.57	13713.03	13729.31	.0012	1
.071240	344.31	.068500	333.70	13526.57	13641.96	13663.14	.0016	2
.079460	600.53	.071240	569.60	13153.84	13205.66	13312.98	.0081	3
.079460	355.53	.071240	325.30	13153.84	13173.71	13281.53	.0082	4
.071240	586.32	.068500	576.90	13372.54	13495.39	13512.44	.0013	5
.071240	341.32	.068500	331.20	13372.54	13443.71	13463.43	.0015	6
.082200	602.26	.071240	562.90	12752.64	12774.96	12939.78	.0129	7
.082200	357.26	.071240	318.40	12752.64	12759.40	12924.76	.0130	8
.400040	1750.14	.405520	1768.40	2244.99	2286.20	2286.72	.0002	9
.400040	1505.14	.405520	1523.40	2244.99	2283.22	2283.69	.0002	10
.400040	1750.14	.405520	1768.40	2244.99	2286.20	2286.72	.0002	11
.400040	1505.14	.405520	1523.40	2244.99	2283.22	2283.69	.0002	12
.402780	1757.84	.402780	1758.80	2243.72	2247.81	2252.61	.0021	13
.402780	1512.84	.402780	1513.80	2243.72	2247.54	2251.94	.0020	14
.402780	1754.33	.402780	1758.80	2241.35	2247.80	2252.45	.0021	15
.402780	1509.33	.402780	1513.80	2241.35	2247.52	2251.50	.0018	16
.010960	388.36	.013700	397.10	2049.39	2727.29	2767.89	.0149	17
.010960	143.36	.010960	142.60	2049.39	2488.96	2499.02	.0040	18
.010960	388.36	.013700	397.00	2049.39	2727.08	2767.89	.0150	19
.010960	143.36	.010960	142.60	2049.39	2488.96	2499.02	.0040	20
.010960	386.70	.008220	377.00	2008.71	2071.47	2115.31	.0212	21
.010960	141.70	.010960	142.60	2008.71	2064.74	2090.89	.0127	22
.010960	383.82	.010960	385.80	1940.68	2067.66	2080.07	.0060	23
.010960	138.82	.010960	139.10	1940.68	2018.53	2018.61	.0001	24
.060280	560.98	.060280	591.80	334.66	401.83	491.43	.2230	25
.060280	315.98	.060280	339.10	334.66	385.08	452.32	.1746	26
.060280	560.98	.060280	591.80	334.66	401.83	491.43	.2230	27
.060280	315.98	.060280	339.10	334.66	385.08	452.32	.1746	28
.060280	560.98	.063020	569.70	334.66	349.57	350.35	.0022	29
.060280	315.98	.060280	315.20	334.66	345.85	346.44	.0017	30
.060280	560.98	.063020	569.70	334.66	349.57	350.35	.0022	31
.060280	315.98	.060280	315.20	334.66	345.85	346.44	.0017	32
.210980	120.26	.200020	116.60	4535.82	4597.42	4605.96	.0019	33
.210980	92.26	.200020	88.30	4535.82	4586.76	4594.58	.0017	34
.238380	124.07	.210980	112.60	4350.28	4375.53	4417.99	.0097	35
.238380	96.07	.210980	84.60	4350.28	4367.76	4410.86	.0099	36
.210980	119.92	.197200	115.30	4516.39	4568.67	4577.77	.0020	37
.210980	91.92	.200020	88.00	4516.38	4559.51	4567.67	.0018	38
.241120	124.28	.210980	111.80	4299.90	4320.17	4369.93	.0115	39
.241120	96.28	.210980	83.90	4299.90	4313.78	4364.03	.0117	40
1.186420	514.21	1.189160	514.80	758.81	765.55	766.15	.0008	41
1.186420	486.21	1.189160	486.80	758.81	765.34	765.94	.0008	42
1.189160	514.64	1.189160	514.80	758.56	765.55	765.94	.0005	43
1.189160	486.64	1.189160	486.80	758.56	765.34	765.73	.0005	44
1.189160	514.95	1.186420	513.70	758.40	758.36	758.59	.0003	45
1.189160	486.95	1.186420	485.70	758.40	758.34	758.57	.0003	46

OPT T DETRM	OPT ORD UP TO DET	OPT T STOCH	OPT ORD UP TO STC	TOT CST DETRM	TOT CST STOCH	TOT CST STC T*R DETRM	PERCENT DEVIATION	ITEM #
1.194640	515.81	1.183680	512.60	757.40	758.36	761.15	.0037	47
1.194640	487.81	1.186420	485.70	757.40	758.34	761.04	.0035	48
.030140	51.77	.032880	52.50	689.19	756.56	773.92	.0229	49
.030140	23.77	.032880	24.60	689.19	738.83	745.93	.0096	50
.032880	52.34	.030140	51.30	682.85	754.83	755.87	.0014	51
.032880	24.34	.032880	24.60	682.85	736.59	737.15	.0008	52
.030140	51.42	.027400	50.20	675.13	659.48	665.70	.0094	53
.030140	23.42	.030140	23.50	675.13	672.94	677.96	.0074	54
.035620	52.52	.027400	50.10	645.82	658.18	680.60	.0341	55
.035620	24.52	.030140	23.00	645.82	666.33	674.02	.0115	56
.178100	111.24	.183580	119.20	113.14	133.20	136.77	.0268	57
.178100	83.24	.178100	88.40	113.14	130.53	133.58	.0233	58
.178100	111.24	.183580	119.20	113.14	133.20	136.77	.0268	59
.178100	83.24	.178100	88.40	113.14	130.53	133.58	.0233	60
.178100	111.17	.178100	110.50	113.10	114.53	115.41	.0076	61
.178100	83.17	.178100	82.60	113.10	114.34	115.09	.0055	62
.178100	111.03	.178100	110.50	113.03	114.53	115.23	.0061	63
.178100	83.03	.178100	82.60	113.03	114.34	114.91	.0049	64

APPENDIX VI

SAMPLE OUTPUT FOR AN ARBITRARY VALUE OF THE OPERATING COST FOR THE Q PROGRAM

OPT Q DETRM	REORD PT DETRM	OPT Q STOCH	REORD PT STOCH	TOT CST DETRM	TOT CST STC WITH OPR CST	TOT CST STC Q,R DETRM	OPERATING COST	PERCENT DEVIATION	ITEM #
235.47	339,97	240.33	340.59	13526.57	13955.51	13860.00	100.00	-.0068	1
235.47	94,97	240.33	95.59	13526.57	13955.51	13860.00	100.00	-.0068	2
246.81	322,42	249.39	322.00	13153.84	13383.37	13284.07	100.00	-.0074	3
246.81	77,42	249.39	77.00	13153.84	13383.37	13284.07	100.00	-.0074	4
235.90	336,98	240.52	337.75	13372.54	13796.18	13700.92	100.00	-.0069	5
235.90	91,98	240.52	92.75	13372.54	13796.18	13700.92	100.00	-.0069	6
247.98	314,56	250.47	314.21	12752.64	12981.13	12881.78	100.00	-.0077	7
247.98	69,56	250.47	69.21	12752.64	12981.13	12881.78	100.00	-.0077	8
1403.12	350,00	1409.50	359.98	2244.99	2371.17	2285.32	100.00	-.0362	9
1403.12	105,00	1409.50	114.98	2244.99	2371.17	2285.32	100.00	-.0362	10
1403.12	350,00	1410.85	354.83	2244.99	2365.09	2267.22	100.00	-.0414	11
1403.12	105,00	1410.85	109.83	2244.99	2365.09	2267.22	100.00	-.0414	12
1404.22	348,11	1409.63	357.34	2243.72	2367.15	2278.72	100.00	-.0374	13
1404.22	103,11	1409.63	112.34	2243.72	2367.15	2278.72	100.00	-.0374	14
1406.25	344,60	1411.55	348.33	2241.35	2355.80	2256.92	100.00	-.0420	15
1406.25	99,60	1411.55	103.33	2241.35	2355.80	2256.92	100.00	-.0420	16
34.16	350,00	41.18	359.57	2049.39	3144.96	3772.64	100.00	.1996	17
34.16	105,00	41.18	114.57	2049.39	3144.96	3772.64	100.00	.1996	18
34.16	350,00	42.80	354.28	2049.39	2924.67	3029.28	100.00	.0358	19
34.16	105,00	42.80	109.28	2049.39	2924.67	3029.28	100.00	.0358	20
35.14	348,34	41.32	357.02	2008.71	3000.45	3458.49	100.00	.1527	21
35.14	103,34	41.32	112.02	2008.71	3000.45	3458.49	100.00	.1527	22
36.89	345,46	43,56	348.27	1940.68	2610.06	2603.70	100.00	-.0024	23
36.89	100,46	43.56	103.27	1940.68	2610.06	2603.70	100.00	-.0024	24
209.17	350,00	213.96	371.89	334.66	477.36	605.16	100.00	.2677	25
209.17	105,00	213.96	126.89	334.66	477.36	605.16	100.00	.2677	26
209.17	350,00	214.45	369.09	334.66	473.66	483.77	100.00	.0214	27
209.17	105,00	214.45	124.09	334.66	473.66	483.77	100.00	.0214	28
209.17	350,00	214.05	369.47	334.66	473.62	530.06	100.00	.1192	29
209.17	105,00	214.05	124.47	334.66	473.62	530.06	100.00	.1192	30
209.17	350,00	214.81	363.86	334.66	465.87	408.67	100.00	-.1228	31
209.17	105,00	214.81	118.86	334.66	465.87	408.67	100.00	-.1228	32
79.73	35,86	85.61	40.66	4535.82	5276.49	5293.45	100.00	.0032	33
79.73	7,86	85.61	12.66	4535.82	5276.49	5293.45	100.00	.0032	34
83.79	28,71	89.12	29.06	4350.28	4791.17	4702.93	100.00	-.0184	35
83.79	.71	89.12	1.06	4350.28	4791.17	4702.93	100.00	-.0184	36
79.75	35.52	85.62	40.32	4516.38	5256.53	5273.55	100.00	.0032	37
79.75	7.52	85.62	12.32	4516.38	5256.53	5273.55	100.00	.0032	38
83.84	27.83	89.14	28.20	4299.90	4740.26	4651.99	100.00	-.0186	39
83.84	-.17	89.14	.20	4299.90	4740.26	4651.99	100.00	-.0186	40
474.62	39.64	479.75	54.85	758.81	891.36	849.01	100.00	-.0475	41
474.62	11.64	479.75	26.85	758.81	891.36	849.01	100.00	-.0475	42
475.12	38.98	480.82	48.30	758.56	882.59	794.84	100.00	-.0994	43
475.12	10.98	480.82	20.30	758.56	882.59	794.84	100.00	-.0994	44
474.72	39.28	479.75	54.50	758.40	890.81	848.50	100.00	-.0475	45
474.72	11.28	479.75	26.50	758.40	890.81	848.50	100.00	-.0475	46

OPT Q DETRM	REORD PT DETRM	OPT Q STOCH	REORD PT STOCH	TOT CST DETRM	TOT CST STC WITH OPR CST	TOT CST STC Q,R DETRM	OPERATING COST	PERCENT DEVIATION	ITEM #
475.42	37.95	480.84	47.33	757.40	881.07	793.44	100.00	-.0995	47
475.42	9.95	480.84	19.33	757.40	881.07	793.44	100.00	-.0995	48
11.78	39.71	18.50	52.84	689.19	1980.46	4497.91	100.00	1.2711	49
11.78	11.71	18.50	24.84	689.19	1980.46	4497.91	100.00	1.2711	50
12.19	39.19	20.31	45.68	682.85	1659.40	2277.69	100.00	.3726	51
12.19	11.19	20.31	17.68	682.85	1659.40	2277.69	100.00	.3726	52
11.89	39.36	18.51	52.50	675.13	1960.70	4447.67	100.00	1.2684	53
11.89	11.36	18.51	24.50	675.13	1960.70	4447.67	100.00	1.2684	54
12.49	38.27	20.34	44.83	645.82	1609.89	2200.54	100.00	.3669	55
12.49	10.27	20.33	16.84	645.82	1609.89	2200.54	100.00	.3669	56
70.71	40.00	75.13	64.81	113.14	259.91	694.66	100.00	1.6727	57
70.71	12.00	75.13	36.81	113.14	259.91	694.66	100.00	1.6727	58
70.71	40.00	75.69	60.14	113.14	253.32	335.58	100.00	.3247	59
70.71	12.00	75.69	32.14	113.14	253.32	335.58	100.00	.3247	60
70.76	39.93	75.13	64.47	113.10	259.36	673.96	100.00	1.5985	61
70.76	11.93	75.13	36.47	113.10	259.36	673.96	100.00	1.5985	62
70.85	39.79	75.71	59.20	113.03	251.85	314.74	100.00	.2497	63
70.85	11.79	75.71	31.20	113.03	251.85	314.74	100.00	.2497	64

BIBLIOGRAPHY

1. Alfandary-Alexander, M., An Inquiry into Some Models of Inventory Systems, University of Pittsburgh Press, Pittsburgh, Pa., 1962.

2. Bourgeois, L. G., "Gross Requirements of Planning and Inventory Control," Production and Inventory Management, 3:4, 38-50 (1962).

3. Browker, A. H. and Lieberman, G. J., Engineering Statistics, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1959.

4. Brown, R. G., Decision Rules for Inventory Management, Holt, Rinehart, and Wiston, Inc., New York, N. Y., 1967.

5. Byrne, W. F., A Study of Inventory Decision Rules for Selecting Items for Stockage Under Budget Constraints, Master of Science Thesis, Georgia Institute of Technology, Atlanta, Georgia, 1972.

6. Churchman, C. W., Ackoff, R. L., and Arnoff, E. L., Introduction to Operation Research, John Wiley and Sons, New York, 1957.

7. Cornacchini, W. J., "Data Processing - Management Tool," Production and Inventory Management, 7:3, 37-48 (1967).

8. Eilon, S. and Elmaleh, J., "An Evaluation of Alternative Inventory Control Policies," Production and Inventory Management, 7:1, 3-14 (1968).

9. Foster, F. G., Rosenhead, J. V., and Siskind, V., "The Effect of the Demand Distribution in Inventory Models Combining Holding, Stock-out and Re-order Costs," Journal of Royal Statistical Society, Series B, 33:2, 312-325 (1971).

10. Gallagher, D. J., "Two Periodic Review Inventory Models with Backorders and Stuttering Poisson Demands," AIIE Transactions, 1:2, 164-171 (1969).

11. Gorenstein, S., "Some Remarks on E.O.Q. vs Wagner-Whitin," Production and Inventory Managements, 11:2, 40-45 (1970).

12. Hadley, G. and Whitin, T. M., Analysis of Inventory Systems, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

13. Hausman, W. H. and Kirby, R. M., "Estimating Standard Deviation for Inventory Control," AIIE Transactions, 2:1, 78-81 (1970).

BIBLIOGRAPHY (Continued)

14. Hicks, C. R., Fundamentals Concepts in the Design of Experiments, Holt, Rinehard, and Wiston, New York, 1964.

15. Hiller, F. S. and Lieberman, G. J., Introduction to Operation Research, Holden-Day, Inc., San Francisco, Calif., 1967.

16. Hilton, J. G., "Some Problems in the Application of Inventory Theory," Production and Inventory Management, 5:2, 71-80 (1964).

17. Hines, W. W. and Montgomery, D. C., Probability and Statistics in Engineering and Management Science, The Ronald Press Company, New York, 1972.

18. Kaimann, R. A., "A Fallacy of E.O.Q. ing," Production and Inventory Management, 9:1, 82-87 (1968).

19. Kaimann, R. A., "Re-visiting a Fallacy of E.O.Q. ing," Production and Inventory Management, 4, 12-19 (1968).

20. Kaimann, R. A., "E.O.Q. vs. Dynamic Programming--Which One to Use for Inventory Ordering?," Production and Inventory Management, 10:4, 66-75 (1969).

21. Kaplan, A. J., "The Relationship Between Decision Variables and Penalty Cost Parameters in (Q,r) Inventory Models," Naval Research Logistic Quarterly, 17:2, 253-258 (1970).

22. Keswary, A. K., "Single Item Inventory Models for Backorders and Lost Sale", Master of Science Thesis, Georgia Institute of Technology, Atlanta, Georgia, 1970.

23. Lindbäck, K. A., "The Economic Consequences of Deviations from Economic Lot Sizes," The International Journal of Production Research, 6:2, 97-107 (1967).

24. McFarland, W. B., "An Accountant's Viewpoint on Inventory Costs," Production and Inventory Management, 6:2, 8-20 (1965).

25. Naddor, E., Inventory System, John Wiley & Sons, Inc., New York, 1966.

26. Newberry, T. L., "Measure of Effectiveness for Selected Inventory Policies Under Stochastic Processes," Ph.D. thesis, Georgia Institute of Technology, Atlanta, Georgia, 1961.

27. Raouf, A. and Warmuth, C., "Probability Demand and Infinite Stockout Cost," A.P.I.C.S. International Technical Conference Proceedings, Nov. 3-11, 1971.

BIBLIOGRAPHY (Concluded)

28. Saundercook, J. F., "ABC Inventory Classification System," Production and Inventory Management, 10:2, 23-27 (1969).

29. Silver, E. A. and Meal, H. C., "A Simple Modification of the E.O.Q. for the Case of Varying Demand Rate," Production and Inventory Management, 10:4, 52-65 (1969).

30. Simone, A. J., "Theoretical Refinement and Practical Implementation of a Traditional Inventory Model," Production and Inventory Management, 8:3, 15-31 (1967).

31. Sinha, B. K. and Gupta, S. K., "Sensitivity of Cost Functions in Inventory Models," AIIE Transactions, 1:3, 252-256 (1969).

32. Starr, M. K. and Miller, D. W., Inventory Control : Theory and Practice, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962.

33. Tuite, M. F. and Anderson, W. A., "A Comparison of Lot Size Algorithms Under Fluctuating Demand Conditions," Production and Inventory Management, 9:4, 20-29 (1968).

34. Whitin, T. M., The Theory of Inventory Management, Princeton University Press, Princeton, N. J., 1953.

35. Whitin, T. M., "Report on an Inventory Management Survey," Production and Inventory Management, 7:1, 27-32 (1966).

36. Wight, O. W., APICS Dictionary of Production and Inventory Control Terms, American Production and Inventory Control Society, Chicago, Illinois, 1966.

37. Zimmermann, H. J., "Periodic vs. Perpetual Inventory Control Systems," Production and Inventory Management, 7:4, 66-79 (1966).